DOCUMENT RESUME

ED 436 347 SE 061 524

AUTHOR	Essary, Catherine; Kari, Amy; Rummelsburg, Judy; Gillan, Connie; Kysh, Judy
TITLE	What Makes a Good Elementary Mathematician? Teacher-Research Report for 1996-1997.
PUB DATE	1998-00-00
NOTE	38p.; "Results of research conducted with Presidential Grant on Education from the University of California Office of the President, 1996-97."
PUB TYPE	Reports - Research (143)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Elementary Education; *Elementary School Mathematics; Evaluation; Evaluation Methods; Evaluation Needs; *Mathematics Achievement; *Mathematics Education

ABSTRACT

This research attempts to look for ways to assess elementary school students in mathematics, to discover what they can do, and what they understand. It aims to establish benchmarks for student progress in grades 1 through 5 and to begin a process for identifying students who are at risk for failure in mathematics. This paper presents each teachers' reports explaining their experiences and findings. Results indicate the importance of both efficiency with skills and good use of problem solving strategies in developing good elementary mathematicians. (ASK)

What Makes a Good Elementary Mathematician?
Teacher-Research Report
for
1996 - 1997

Rio Vista Elementary School
by
Catherine Essary, Amy Kari, Judy Rummelsburg,
Connie Gillan, and Judy Kysh

Table of Contents

Introduction 3

Roaming the Known and Recovery

Part I: Catherine Essary:
- Roaming the Known with Darius 4
- Working with Third Grade Girls 5

Part II: Amy Kari:
- Why I Chose Michael 8
- Roaming the Known with Michael 9
- Working with a Group 10
- Results 11

Part III: Judy Rummelsburg:
- Roaming the Known with Jennifer 12
- Group Work 13
- Sequel 14

Part 1V: Connie Gillan:
- Why I Joined the Math Research Group 14
- Who I Chose First and Why 14
- Roaming the Known and Group Work 15
- What Did I Learn? 18

Part V: Judy Kysh:
- Counting All the Cards: Jeremy's Question 18
- Categorizing Student Responses 19
- Comparing Student Responses by Grade Level, Fall and Spring 23
- What Did We Learn? 27

Conclusion and Plans for next year 29

Appendix A
- Fall & Spring Assessment A-1
- Timed Addition Test A-2
- Timed Multiplication Test A-3
- Kamii Test A-4
- Place Value A-5

Introduction

Results of research conducted with Presidential Grant in Education from the University of California Office of the President, 1996-97.

Researchers:
Catherine Essary, Connie Gillan, Amy Kari, Judy Rummelsburg,
Teachers at Rio Vista Elementary School
Judy Kysh,
Education Specialist, CRESS Center, University of California, Davis

In the fall our goal was to look for ways to assess students in mathematics, to discover what they could do and what they understood. We hoped to establish benchmarks for student progress grades 1 through 5 and begin a process for identifying students who were at risk for failure in mathematics. In order to improve our data collection procedures from the previous year's research, we identified a series of assessments to be given as pre- and post-tests. One area we were particularly interested in was the development of multiplicative thinking.

We also wanted to continue looking at the role of speed and efficiency in developing able elementary mathematicians. The assessment plan we developed is discussed in this report and is included in Appendix A.

The assessments were administered by the four teachers participating in the project as a combination of whole class tests and individual interviews. Grant money was used to release the teachers to conduct the interviews. Students came from two 1/2/3 multiage classrooms, a straight third grade and a 4/5 multiage class. The assessments, with the exception of the new one discovered in November, were started early in the school year and completed by November 1.

When we met to discuss the outcomes of the initial assessments we were disappointed to learn that they did not give us the information we expected. In the primary grades the tests helped to identify students performing at a very high level. However, we didn't learn anything new about what our average and low-performing students did know — they simply performed at the levels they perform at on any assessment they are given. We knew a lot about what they didn't know. In the upper grades, the assessments served to separate the average to proficient mathematicians from those who were not; however, as with the lower grades, they did not give us any detailed information about the capabilities of the low-performing students.

What we came to realize was that even though it was nice to know who was making progress and who was successful, our purpose in doing the research all along has been to effect real change for the students who were not successful. We came to the conclusion that the tests we had selected were designed to measure the success of a math program, not of individual students. They did not hone in on individual capabilities. This led us to discuss options for more specific assessments.

With the understanding that there are many differences between learning to read and constructing logico-mathematical reasoning, we looked to Reading Recovery® for some ideas. We consulted our Reading Recovery teacher leader, who suggested that before you can "find out what's wrong with a kid, you have to find out what's right first." In Reading Recovery they call this Roaming the Known. This is an open ended process where the teacher follows the students' lead and explores to discover all she can about what the child already knows. This idea intrigued us and we decided to adapt it for mathematics. Each of us selected a single low-performing student to work with individually for half-hour session four times a week for a minimum of eight times. We video-taped many of these sessions. Following these sessions the group met to discuss their findings. Each teachers' report follows starting with first grade.

Roaming the Known and Recovery

Part I

Catherine Essary

Roaming the Known with Darius

As a first grade mathematician, Darius puzzled me with what seemed to be inconsistencies. I was still asking, "Why can Darius do Stones to 3 and can't play Go Fish?" The Stones assessment is based on Kathy Richardson's work (1984). For the Stones assessment we show the students a number of stones, say 5. The student counts them, then I hide some and show the remaining stones. A student has "mastered 5 stones" when he can instantly say how many are missing. Go Fish Presents is a similar problem with playing cards. The student is aiming for a particular number, say 5. He draws a card, maybe a 4, then has to ask for the card he needs to make 5.

Darius is a six year old African-American first grader. Initially, he seemed to perform in ways similar to other low to average performing first graders. On the first Stones assessment in Sept. 1996, he did Stones quickly and easily to 3. He used his fingers for 4 and 5 and was completely baffled by the sixes. Of ten first graders in his class, two others performed at about this same level, three performed at a lower level and three more at a higher level.

Also in September, I gave a 10 minute timed test of 100 addition problems with sums to 18. Of the first graders, there was one score of 30 and the other scores ranged from zero correct to 14 correct. Darius got zero. On the "Just Right Problem," students were asked to write some math problems that were just right for them. First grade responses ranged from simple addition (i.e. 1+1, 5+5, 10+10) to a list of numbers, a list of words, and scribbles. Three first graders did simple addition problems, three wrote lists of numbers, one wrote numbers and letters together, one wrote just letters and two scribbled. Darius scribbled.

When I ranked the class according to problem solving abilities, in November, I ranked Darius fairly low at 16/20, with 1/20 being the top and 20/20 being the bottom of the class. In February, however, I ranked him at the very bottom 20/20 where he remained for the rest of the year.

As we started developing a program based on some of the aspects of the Reading Recovery® model, I knew that Darius was the student that I had to take to Roam the Known. By all our measures at that point, his performance ranked as the lowest or one of the lowest in my class. By February, 1997, it appeared as if he had made little or no progress in mathematics. At the same time, he had also been selected for Reading Recovery as one of the lowest achieving first graders in reading at our school.

As with the Reading Recovery® model, our model began with sorting out what Darius actually did know about mathematics. During February, 1997, I spent four half-hour sessions with him trying to find out. I redid the stones assessment and to my surprise, Darius could do stones to 4 without using his fingers. When we started Stones he said, "Are we gonna play with 8?" I said, "No." He said, "Four then?" I said, "O.K." I hid 2 and said, "How many are hiding?" He said, "Two. Yeah!" and gave me a high five! He said he wanted to do the hiding. He grabbed a bunch of marbles and closed them in his hand and said, "How many are hiding?" I said I couldn't tell until I knew how many we had started with. I asked how many he had, and he looked in his hand and said 5. Then he hid 3 and asked the question again. When I said that I could see 2 so there must be 3 hiding he said, "No. There weren't." Then he wanted to stop playing.

He could rote count to 15, and was able to make 1:1 correspondences. He did not conserve. He knew the doubles 1+1 and 2+2. He knew 3+3 and 4+4 one day, but the next day counted dots on the dice. He also knew 1+2 and 4+2 without counting dots. In all the other cases, he counted dots on the dice. I observed that he didn't always start counting with the bigger number, nor did he count on. He knew which dice had the bigger number and which one had the smaller without having to count the dots.

On one occasion, I pushed a big pile of cubes his way and asked him to divide it fairly. I asked if that pile was fair. He said that it wasn't. I pushed some towards me and asked if that was fair. "It isn't," he said, "but I want them all." He pushed some more around and said again, "I want them all." "Is that fair," I asked. "No. But, I want it." He started to line his pile up and said, "I want these." I asked, "Do you want most of them or all of them?" "I want most of them." "Is that fair?" "No," he said. I asked if it would be okay if we had the same amount. He said okay. He finally began to count the cubes and made two even piles of 11, for each of us.

Another time we were looking at a book with different numbers of things on each page. When I asked which page has more, the socks or the shoes? He correctly answered shoes. When we got to the page with flowers I asked which page has more flowers? (One page had five and one page had 2 for a total of 7). He counted 7 flowers. I asked which page has more flowers? He pointed to the page with 5 flowers. Then I asked which page has less flowers. He looked at me with a puzzled look, shrugged his shoulders and tentatively pointed at the page with 2 flowers. It could be that he was wondering why I was asking such a silly question, which his previous answer made obvious. We had read excerpts from Heath's (1983) research reported in *Ways with Words* in which she followed pre-school African-American children as they were growing up at home. She noticed that children were not generally asked to respond to simple questions with obvious answers, questions for which they knew the adult knew the answer. Whereas White children in similar circumstances were often asked to respond to such school-type questions.

We were going to play the card game War which Darius calls "Pairs." He offered to "deal " which consisted of giving me a bunch of cards and he took a bunch. I asked if that was "fair." He just stared at me. We began by each putting down a card. Darius said "Oh goody, I won." When I asked why he won, he said, "Because I want it." We played along for a while with him randomly giving cards to me and to himself as having been "won." Then he stopped and looked at his pile and said, "I won cuz I gotta lot of cards." I asked who had more, he or I. Without counting he said, "Me, cuz I won." When we played "Fish" I ran out of cards so I said I won. He said "No, take some of my cards."

As we were looking at children to select for group work, I decided not to choose Darius. It was clear, as we evaluated him, that his skills were much below the average first grader. In fact, to several of us, it seemed as if he worked more like a four year old because he didn't know and use rules for games and had few strategies for problem solving. Amy did take him for a try out in her group, but as he worked with the others, he often did not respond to the tasks at hand, but gave random answers or just played with his cubes.

At this point I began wondering if Darius was confused about math, the language of math, or language in general. I knew a lot about what he knew, I knew a lot about what he didn't know. I felt like I didn't know enough about what to expect him to know. What does the average first grader know in mathematics and what is reasonable to expect? I went back to the research group with more questions than answers.

Working with Third Grade Girls

Our schedule of assessments called for us to re-administer the Kamii (1985) assessment to the third graders. (This set of exercises is in Appendix A.) By this time, all of our third graders had been moved to an all third grade class. We planned to identify the "at risk" students so that we would have time to work with them for a while before the school year ended. In looking at students for the group, I looked at this test which is given during an individual oral interview (Scores in Table 1). Not only does it produce a numerical score, but anecdotal notes as well. The beginning portion of the test is timed. We also looked at the student's ranking on the Problem Solving Ranking list.

Kamii Scores - possible 45			
Name	**Spring '96**	**Fall '96**	**Spring '97**
Thien		25	34
Kathryn		30	34
Maria		30	22
Jennifer	37	33	36
Darla		33	35
Samantha		38	37
Alexis	29	39	35
Lilly		41	39
Leila		31	31
Aida		31	35
Latisha		28	33

Table 1

Reading Recovery® takes students with the lowest of the low scores. In this case, we made the decision to take students who were closest to but still below grade level. We thought that they would benefit most by short term intervention.

The two I felt the most concern for were Jennifer and Alexis. I have followed both these girls for two years and watched their sense of mathematics develop. During this screening both girls seemed nervous and kept asking "Is that right?" They both used the standard addition algorithm and made mistakes. They both wanted to use pencil and paper. I told them that they were supposed to use mental math to figure them out. They said they could, but didn't want to. Jennifer said, "I can't do math anymore." Alexis said that she didn't have any confidence and anyway other people did math better than she did. I commented to Alexis that she did her single digit addition "facts" very fast now, she said surprised, "I do?" Both girls had been very quiet during mental math early in the fall, but they had just begun to participate in the process. When they were moved into an all-third grade class, they both said that now they did not participate in mental math at all. I asked why. Jennifer said that "...the boys were louder and that the teacher would listen to them give any old dumb answer." I asked if she had ever had ever challenged their answers she said, "No, I just sit there and be quiet." Alexis said that she often knew the answer but wouldn't raise her hand because Isaiah and Angelica are smarter. When I asked what that meant she said, "You know, they do it really fast."

Maria's scores were very low. I did not select her for group work because as I interviewed her it appeared that she had some serious misconceptions about number. When she added a two-digit number to another two-digit number, she just added all the digits together without regard to their place value. This resulted in a discussion with the speech therapist about whether Maria might have a language processing problem or some other learning problem. She is being screened for further testing.

Amy selected Aida, Latisha and Leila for recovery because during the screening all three seemed hesitant, and not as engaged in the mathematics as they had been just a few months before. All tried to use standard algorithms to solve the problems and used them incorrectly. They seemed to have lost their mental math strategies.

Results of the Group Work

I met with these five third grade girls three times a week for half hour. We spent the first 15 minutes playing board or card games and the last 15 minutes doing mental math.

During the first meeting I asked the girls why they thought they were selected. Latisha said, "Because we're dumb in math." I asked the others if they thought so, too. They agreed. I taped the third meeting during which we had a conversation about math. The girls agreed that they used to be good at math, but as Latisha put it, "Now I'm losing it." When asked why or what happened. Latisha said, "I used to go oh, oh, I know the answer. But now I just fiddle in my desk." Leila said, "Well we used to have these cubes and beads and stuff to use." This resulted in a lively amount of chatter agreeing that that was one thing that they really liked in their math classes before. Jennifer said, "What I really liked was playing those board games. They were really fun." She mentioned that her current teacher only had a few games so that not everyone could play together. Leila and Alexis wanted to learn more card games. Both of them said they had no one to play with at home. Leila said, "When I go to sleep I always dream that my sister will play math games with me, but then I wake up and it never comes true."

As I watched the tape later, it occurred to me that maybe these girls had not lost their math skills, but rather had lost their confidence. The social interaction seemed important to these girls. I used these ideas in planning the sessions. I wanted them to see math as fun again and that they were not "losing it." Several times I brought new board games for them "to try out." They gave me advice on what rules there should be as they played with each other. I made a set of these games and gave them to their teacher. Other times I taught them new card games that they could play by themselves at home. Alexis said that she taught one of them to her little sister who is in Kindergarten. Jennifer said she tried to teach her little brother but he wouldn't play by the rules she taught him.

During mental math time we discussed strategies and the "best" way to do a problem. One day we had a discussion about the standard addition algorithm. Jennifer, who had been in my class for two years, was not taught the algorithm during math instruction. She was always encouraged to invent her own procedure. Now I noticed she was using the algorithm. I asked her where she had learned it. She said, "That's the way we do mental math." I asked if it was better than her own procedures. She said she didn't know but that was the way they were supposed to do it. I pointed out that her answers were almost always wrong when she used it. She didn't seem too surprised to hear that. She said it didn't matter because her teacher would come by and pat her on the shoulder and tell her to try again. I asked what happens if she gets it wrong a second time. She said, "He says, 'well at least you tried.'" The girls just laughed and said yeah that's what he does. So we made a little rule that for mental math with the group they had to use a procedure that wasn't the algorithm. At first, Alexis especially had trouble doing the math "in her head." Later she got very fast and could do some of the problems in multiple ways. In a discussion with Amy, Aida's former teacher, I mentioned the Aida was always chiming in with mental math answers and strategies during group time. She was surprised because she said that Aida hadn't participated in mental math when she was in her class.

We were not able to do a post test with any of these girls so I do not have numerical evidence that there was improvement. What was evident is that the girls enjoyed math when they were in the group. When I saw these girls in the hall they would invariably ask, "Is there group today?" They always expressed positive feelings about being in the group and wanting to participate. The extra attention did contribute to a more positive attitude about math. All the girls said they liked the group but they weren't sure if it helped them when they went back to their regular class; although, they were very pleased that their teacher had more games in class for them to use.

REFERENCE

Richardson, Kathy (1984). Developing Number Concepts Using Unifix Cubes. Addison Wesley Publishing.

Part II

Amy Kari

Why I Chose Michael

A year ago last spring, I was asked by our ESL teacher to take Michael the following year as a first grader in my multi-age program. Michael was described as a likely candidate for special education. He was described as having large and small motor problems.

When Michael arrived in my room in September he was still five years old as were several of my other first graders. Michael is Vietnamese, he lives with his mother, a step father and some siblings. His mother was in her mid-teens when Michael was born. She speaks fairly fluent English and is going to school. She is very concerned about Michael. Although the school testing showed that Michael was performing at such a low level that he was a candidate for special education, I pushed for him to participate in Reading Recovery®. Our ESL teacher worked with Michael as a Reading Recovery® student starting in mid-September.

In the classroom, Michael began the year ranked near the bottom as a problem solver (ranked 19 out of 20 with 1 being highest) and as one of the two lowest scorers in all other areas. When interviewed in mathematics in September, Michael could count ten objects, beyond that he called out numbers randomly and without one-to-one correspondence). Michael could do stones to three but was unable to do anything with four. He did not conserve number. (This was not unusual, only one first grader student did). Michael scored zero on the timed test as did one other first grader. The other first grade scores ranged from 1-42 out of 100 correct. Michael had difficulty forming numbers and letters legibly, most of his work appeared as random scribbles. He had trouble focusing during math time and wandered around instead of playing the games introduced in class. Whenever possible I would have Michael and other low scorers play games with the special education assistant who came to my room two half-hour periods a week. Sometimes she worked with Michael on number writing.

Michael's relative status in the classroom remained unchanged throughout the fall and early winter, but he did become more engaged in game playing at a very basic level. He loved to play a version of dice war with his friends (also low performers) that included rolling two dice, counting the dots and then building a tower of unifix cubes to match the count. Each tower was added on to the prior one resulting in very long trains of cubes which the boys compared to find a winner. Interest in this game remained high through most of the year. Michael usually chose to play this game instead of working on class problems. During this time period, Michael struggled with all aspects of learning. After making initial progress in Reading Recovery, Michael stalled at a very low level and was unable to move further. Ultimately, he was dropped from the program for lack of progress and moved to a small ESL reading group for the rest of the year.

When the time came to select a student for "Roaming the Known" for math research, I had to choose between Michael and two other students who were performing at about the same level. The other two boys, one Latino and one African American, were both Reading Recovery® students.

Manuel was moving along nicely with his reading and I didn't really want to distract him and pull him from the classroom any further. Cleveland, like Michael, was not doing well in Reading Recovery and had recently been diagnosed as ADHD but his mother was resisting making any decisions about treatment (whether medication or some other method).. Cleveland was extremely difficult to work with even one-on-one. Given this I was already leaning towards choosing Michael when I began to notice evidence of change in his performance. In late January, I observed Michael playing dice war with a second grade girl, instead of his usual partners. Michael was easily rolling two dice and counting accurately to get an answer. Once when Courtnie rolled a five and a one, Michael called out "six" before she did. He did the same for the combination four and one, while Courtnie still had to count the dots. I was intrigued-- WHAT DID MICHAEL KNOW?

Roaming The Known With Michael

I worked with Michael for seven sessions about twenty to thirty minutes in length. He was really excited to be going with me and after the second session began asking each morning if we were going to "go to math" again. During the first session I noted an immediate change from Michael's performance in September. Michael was able to handle the combinations of Stones to five with relative ease-- there was some slight hesitation accompanied by eye movement as he checked the number of stones in my hands. When we moved to six stones he made some errors but self corrected on some of them.

When I asked Michael to get ten stones, he counted them out one by one, stopped and checked his count and then said, "OK." When I added one stone, he said, "Eleven," then "Twelve" for the next one. When I added the thirteenth stone, he paused for a long time. He recounted the stones in his head, lips moving silently and stopped again. I eventually gave him the word thirteen. Over the next several sessions, it became clear that Michael could count accurately to twelve, but no further. He did not even remember the name "thirteen" during all of our sessions. When given thirteen, he would randomly name other numbers like "14, 16, 19, 14, 15..." It was interesting to note that when given a pile of around twenty cubes, Michael would routinely estimate that there were twelve. Michael was also able to write the numbers from 1-12 with little hesitation except when he monitored several numbers he reversed.

When Michael and I played with the dice, it turned out he knew his doubles quickly and confidently. He also knew combinations of any number and one. For example, once when I threw five and one, he started to count and then stopped and said "ooh, six!" When combinations like five and four came up, Michael counted the dots sometimes from a distance, sometimes by touch. He always started at one. Although he could handle the dice combinations so competently, Michael proved to be very confused by the symbolic representations of addition. He could not add the five and a one on the cards, and he easily confused the symbols + and = when asked to record equations.

During one session I gave Michael some cubes representing cookies. He counted and found there were ten, then easily divided them into two groups of five to share them with me. I asked him how many more cookies he would need to get so we would have twelve altogether. Without hesitation, he said "two more." When I asked how he knew that, he said, "Cuz I counted it before." During this same session I showed him two cards, a six and a five and asked how much that was. Again Michael quickly and accurately responded without counting. But some combinations he counted the shapes always starting at one. Finally, I showed him a seven and a six. Michael began to count. When he got to twelve he stopped, his finger hovering over the thirteenth shape. "What's that name," I said, "Do you remember?" Michael responded no and I told him thirteen.

I was continually amazed at how much Michael knew about number to twelve. When we discussed what Michael knew during our research meetings I was very excited about how this student I had seen as so far behind, knew so much. However, I was also puzzled by the limits of what he knew, in particular his inability to move beyond twelve. Two questions kept recurring in my mind: HOW WERE WE GOING TO BREAK THROUGH TO THIRTEEN? and DID MICHAEL KNOW MORE OR LESS THAN OTHER FIRST GRADERS?

Working With A Group

When the research group got together to discuss our roaming experiences, we were all in agreement that we needed more information about what other students at the same grade levels were like. We decided to "roam" with some more students at each grade level in a more abbreviated fashion to try to establish some benchmarks for high, medium and low performance. Catherine and I set out to interview the remaining 18 of our first graders.

After completing these interviews it was apparent that Michael was still one of the three lowest performing students. However, it was interesting that Michael's ability to work with numbers up to twelve put him within the range of the average. It was only his inability to work with larger numbers that separated him from the average group. I wondered what this meant and whether or not it would be possible to provide Michael with the kind of stimulation that would move him into the average group.

At the next session of our research group when we discussed the idea of moving from "Roaming the Known" to group work, I was concerned that working with an individual student as is done in the Reading Recovery model would not be most effective. The source of this concern was the reading I had been doing (Kamii 1985, pp. 26-36) about the role of social interaction in the development of logico-mathematical thinking. Kamii writes, "Insofar as peers and adults constitute the child's social environment and the objects of his social interaction, they influence his construction of logico-mathematical knowledge in very important ways. They fuel the child's mental activity by such indirect means as saying something that casts a doubt in his mind about the adequacy of an idea. They also do things that become for him an impetus for making a new relationship."

I knew from personal experience that it is very easy for an adult to push and explain too much to a child rather than simply casting doubt and providing an opposing point of view. I suggested to the research group that we work with small groups of children who were roughly at the same level of performance. The idea would be to provide the small group with problems or situations that were likely to generate a variety of differing viewpoints thus stimulating social interaction and growth.

Catherine and I decided to put together a group of four of the lowest performing first graders, two boys and two girls, and I planned to work with them three days a week for five weeks. Michael was included in the group. By the third session I had expanded the group to six students and when students were absent decided to pull in "substitutes (always students performing at the lowest level of the classes)." This last decision proved to produce very interesting results. The students and I would go to a small room and work together. We video-taped most of these sessions.

Although I did many activities with the group each session, one task produced the most interesting discussion among the students and resulted in the most obvious change in thinking. The task consisted of my making two trains of unattached unifix cubes equal in length lined up in one to one correspondence. The students counted with me as I made the trains and I always confirmed the number of cubes in each train before pushing the trains in opposite directions and asking "Which one has more?"

The first time I did this (six cubes in each train), there was an immediate response. Cleveland said, "This one" and pointed to a train. Lokelani simultaneously said, "That one" and pointed to the opposite train. From there the discussion became noisy and vigorous. Mary was the first to say, "They have the same." Her comment which she backed away from when challenged by the others, led to counting, recounting, and debate about what I had asked. It went on for about ten minutes. These students were engaged as I had never seen them before. Each of them changed their point of view several times during the discussion. I facilitated the conversation and occasionally repeated my original question "which has more?" and took them back to the original number of cubes in each train. I also clarified language like more, same, different, and both which they used in many contexts. At the end they came to agreement that the two trains were the same.

By session three, the group had expanded to six, and I decided to replace Mary with Jackie because Mary's verbal skills were so far superior to the others that she out talked them. She also caught on to the concepts so quickly that the others lost their think time. When the same task was repeated with eight cubes, all the students immediately chorused "the same." Only Cleveland physically counted the cubes to check his response. Session five with the same students and ten cubes in each train was more similar to session one. There was more debate although they came to agreement much more quickly. I continued to include this task in our daily sessions, changing the number of cubes.

Ten days after session five, something interesting occurred. Samantha, a student who was an excellent reader and writer but a less confident mathematician, came in as a substitute for Cleveland. I assumed she would have no problem with what I now called the "conservation task" with the cube trains. However, when I placed the two trains of ten cubes on the table and pushed them in opposite directions, Samantha pointed to one as having "more." The group immediately indicated their disagreement with her position and proceeded to attempt to persuade her to their point of view. What was notable to me was the uniform improvement in their explanations. Their explanations were clear and concise: "They are the same because they both have ten, see watch me count." "If Mrs. Kari put one more here then it would have more."

Startled by Samantha's response to the conservation task, during the remaining sessions, I made the decision to keep pulling in new students when I needed a substitute. Each time a visitor arrived, they pointed to a train as having more and each time the group worked to change their view point, continuously improving the clarity of their responses.

Results

At the end of my small group sessions in late May, I decided to reassess my first graders to find out how far the class had come since the fall and to check the performance of the students who had participated in the group compared to their classmates. I administered three assessments: the Timed Test, the Conservation Task, and Stones. Everyone showed significant progress from the fall. The range of scores on the Timed Test rose from 0-42 to 11-100, with seven of the ten students scoring above 50. Mary and Jackie two participants in the recovery group raised their scores from 2 and 1 respectively to 52 and 75. Michael moved from 0 to 36 despite continued difficulty with fine motor skills which effect his ability to write quickly. Six out of the ten first graders, including Michael, had Stones to six fluently while three of the remaining four had strategies for getting six accurately. Only Manuel made some errors which he could not correct. All of the students from the group now demonstrated proficiency on the Conservation Task. Michael said, "They are the same--both tens." Cleveland told me, "You trying to think this one is bigger because it's in front, but they are still both the same. If you put this here (he moved the rows opposite to the way I had them), they still the same. They both nine." In fact, the only student in my first grade group who did not pass the Conservation Task was DeNisha who also happened to be the only student who did not attend a group session as a substitute and who had not conserved number in the fall. My exclusion of DeNisha from the group sessions was as accidental as my inclusion of Samantha had been, and she had always seemed to be a very competent first-grade mathematician. When Catherine screened her first graders they all had conservation with the exception of two students. One of these students Darius was in general performing at a low level, but Steven, like DeNisha, was, otherwise, a very competent first-grade mathematician. Darius had attended the group as a substitute and had trouble keeping up. DeNisha and Steven had never attended. Clearly the failure of Steven and DeNisha on this one task does not prove the success of the groups, but combined with the improvement seen in the group population in general, I feel very good about the initial result. For Michael, in particular, the individual and small group interactions seem to have resulted in the solidification of shaky concepts and provided an opportunity for him to build new relationships about numbers at an accelerated pace. By the year's end Michael was easily counting up to twenty-nine and was beginning to construct the pattern of numbers over twenty. The question of how to break the barrier of thirteen was answered during the small group sessions when the students were arguing about how to count a pile of cubes.

For the future, I wonder about the opportunities for early identification of problems based upon finding out what an individual student really knows. I wonder about the possibilities for modifying classroom instruction to provide for small group interactions around a simple task — just challenging enough to generate conflicting view points that beg for resolution. Can we design a program to recover at-risk elementary mathematicians before they get too far behind in the upper grades?

Part III

Judy Rummelsburg

Roaming the Known with Jennifer

Jennifer is an 8 year old third grader. She comes from a two parent home with a limited income. She is an active student, interested in learning but impatient. As a result, she makes silly errors. There seemed to be no external reason for her low performance in math. In the fall, Jennifer scored 28 out of 43 problems on the Kamii assessment. Out of a class of 31, four other students scored below her. On the fall Stones assessment, she initially answered how many stones I was showing, not how many I was hiding. I explained the task again until she understood. On Stones to six, she was slow in responding and used her fingers constantly. I was concerned because this was an assessment used to screen first graders.

I chose Jennifer for "Roaming the Known" for several reasons. In the fall, she had low scores on all of the assessments and yet she seemed capable of more. She attended school regularly, had a positive attitude, was willing to take risks, and often participated in mental math. I also chose her because she was a girl. I've noticed that some, girls at this age begin to lose confidence in math and science and slip behind their male peers. I worked with Jennifer for seven half hour sessions.

Initially, we played two card games that she had learned in class, Fishing for 10's, and Aces Out. "Aces Out" is like fishing for 15. The aces are left out of the deck and the J, Q, K count as 11, 12, 13. Then I got out six probability chips. She would shake them and throw them on the table and record the math equation (i.e. 3 yellow chips + 3 red chips = 6 chips). I had her use the chips for sums to 5, 6, and 7. The task seemed fairly easy so I asked her what she thought was hard. She replied, "adding 3 cards together is hard for me. It's also hard when you put the problems in a row [5 + 5 =] instead of on top of each other ." I gave her the problem 75 + 37. She answered it by using the standard algorithm but could not explain her answer. This led me to questions concerning her knowledge of place value. When I asked her what the 3 meant in the number 37, she replied, "30" but, when I asked her to solve the problem "the other way" (75 + 37), she could not do it. I thought that she might not fully understand place value.

After talking to members of the research group about what was happening, I decided to go back and explore the low level (i.e. sums to 10 or less) math facts and to try and devise more meaningful assessments. I had Jennifer go back to low-level math games. She started with rolling two dice and adding them together. It became apparent that Jennifer knew her doubles combinations. We played "Aces Out" (Fishing for 15) and she was immediately slower and used her fingers. We shifted to "Fishing for 10's." She was faster and some of the sums were more automatic. Next, we played "Domino War" where each player flips over a domino, adds the dots and whoever has more, wins. Again, it was apparent that she knew the doubles combinations. She also knew which person had more and which had less. I decided to rescreen her on Stones and found she knew Stones to 4.

One of the most interesting things I discovered about Jennifer was her ability to estimate. After playing Domino War, I asked her how many total dots she thought there were on all of the dominoes. She said, "100." The actual total was 103. We played again with only some of the dominoes. Again, I asked her how many total dots there were. She replied "80." The actual total was 80. The following day, I dumped all of the probability chips on the table and asked her to estimate. She said "200." The actual total was 200!

I do not know if the skill of estimation is important in third grade. I also don't understand why she can estimate but has trouble with math facts to 10. After roaming the known with Jennifer, I learned that I needed to work with a larger sample of students to get a better idea of what skills third graders had and which skills were necessary for third grade and to move onto higher-level math.

Group Work

My experiences with Jennifer were similar to the other teacher's experiences with their students. We were excited and puzzled by what we saw and wanted more information. We came to the conclusion that we needed to know what skills an average first grader had, what it looked like for an average second grader and so on, up to fifth grade. By "Roaming the Known" with several students and refining our assessments, I was able to determine which students were below-average, average, and above-average mathematicians. Jennifer appeared to be a below-average mathematician. There were several students in this category. I chose three other female students (Thuy, Kristin, and Evanjelina) with a similar profile to Jennifer's. Each of the girls came from a two-parent home with a limited income. They attended school regularly and were active participants. By external appearances, the girls should have been performing at a higher level than they were. For example, in the fall Kamii assessment, Evanjelina and Kristin scored in the high 20's. Thuy scored 40 out of 43. However, each of the girls only had Stones to 4 and did not understand place value. All were familiar with the standard addition algorithm procedure but didn't understand what it meant. The recovery group met for five weeks, with half hour sessions three days per week.

It was apparent from the very first session that regardless of the mathematical outcome, there would be a very high affective outcome. The girls were extremely animated and almost "silly" during every session. They were motivated to go and often wanted to go on "off" days. Their participation in whole-class mental math problems increased dramatically.

Since each girl had only mastered Stones to 4, we started each session with working on sums to 5 or doubles. The girls each had one die and worked in pairs. For the sums to 5, one of the girls would roll her die then they would try to be the first person in their group to say how many more they would need to make 5. For example, if the student rolled a 3, they would answer 2 (3 + 2= 5). If she rolled a 6, the girls decided to answer "minus 1." Similarly for the doubles, one of the girls would roll her die, and they tried to be the first one to double the number shown and say the answer. Next, they rolled two dice and just added them together. Again, the goal was to be the first one to answer. Many times, I heard the girls defending their answer to their partner. "Nah huh, it has to be 6 because look, 4 plus 2 more is 6. Four, five, six" (using their fingers to illustrate the point).

We ended each session with a mental math problem. I tried to manipulate the problem to allow the girls to use the facts they were learning. For example, one math problem was 6 + 7= ? Kristin answered, "I know 6 + 6 =12. Twelve plus 1 more equals 13." Evanjelina answered, "I know 7 + 7=14. Fourteen minus 1 equals 13."

At the end of all our sessions, I rescreened each girl. All of the girls had Stones to 6 without any hesitation, errors, or use of fingers. On the Kamii assessment, Jennifer improved from 28 to 36 correct answers. Thuy moved from 40 correct to 41. Kristin increased her score of 28 to 37. And Evanjelina moved from 29 to 36. Each girl seemed more confident in her abilities as a mathematician and took more risks.

Sequel

The cards task described in Part V provided further evidence of the girls' confidence and progress. In the fall, Jennifer had added a bunch of tens then added random groups of numbers to get 354. In the spring, she grouped by tens, all the 9 + 1, 8 + 2, etc. Although, for some reason she skipped one of the combinations of 7 + 3, she recovered the 7 and get 337 instead of 340. The fact that she used a much more organized strategy that could lead to multiplicative thinking is promising. Evanjelina's work is shown in Part V, Figures 3 and 4. In the fall, she just wrote down all the numbers randomly from the cards then added the whole column. In the spring she used multiplication with subproblems. Thuy also used multiplication to solve the problem in the spring. Only Kristen was still using a card based strategy of adding randomly drawn pairs then totaling the results. In the spring she did, however, persevere and came close with only a few errors and a result of 334.

<u>Part IV</u>

Connie Gillan

Why I Joined The Math Research Group

I first joined the Math Research group because I was frustrated with my classroom instruction of math. I had wonderful units that I was teaching, but no matter how well the units seemed to go, there were always students who seemed to fly through the material, while others didn't "get it" no matter what I did.

I knew the more proficient mathematicians could be engaged with extensions, but I was uncomfortable with my modifications of the units for the less accomplished students. I tried to simplify the units, but the students still seemed unable to grasp the concepts. Clearly I needed a different approach, but wasn't sure what it should be. I began meeting with Amy, Catherine, and Judy and was stimulated by the discussions and excited by the learning opportunities the group could provide me.

Kid-watching is one of the best ways to see what and how students learn, and the grant gave me a unique opportunity to observe my students extensively. The information I have gathered has given me great insight as to how my students learn mathematics, and how I can better provide a learning environment that supports their growth.

Who I First Chose And Why?

The first student I worked individually with was Farida, a fifth grader whom I thought was functioning on about mid-fourth grade level mathematically. She was very strong in her additive thinking, but seemed to be struggling when it came to multiplication.

I met with Farida in five 30 minute sessions. We usually began with some mental math problems and then proceeded to word problems during which Farida would look for solutions using whatever methods were most comfortable for her.

On mental math addition problems, Farida was comfortable separating tens and ones, and then adding them together. For example, in the problem 35 + 15 + 22, Farida would add 30, 10 and 20 to make 60, and then add 5 and 5 to make 10. She added the 60 and 10 to get 70, and then she added the 2 to make 72. She made good use of looking for 10's first.

In the subtraction problem, 2,083 - 192, she began to use the standard subtraction algorithm. When I asked her to explain the steps, she said, "3 take away 2 is 1; 9 take away 8 is 1; You can't take away 1 from 0, so you have to go next door and borrow some from the 2. That makes the 0 a 10, and the 2 a 1. Ten take away 1 is 9, and you bring down the 1. The answer is 1,911." When I disagreed, she rechecked the problem and became confused, unable to find her mistake. She had inverted the 90 and the 80 to get one. She clearly was uncertain of how to deal with place value in this context.

When doing multiplication problems, Farida chose to use additive strategies. For example, with the problem 35 x 3, Farida added 35 + 35 to equal 70, and then added another 35 for the answer of 105. She did not choose to use the standard multiplication algorithm. When given story problems, she was equally efficient with her addition, but did not apply multiplicative thinking in her solutions.

When asked to solve the problem, "There were 10 people going to the movies, and it cost $7.00 for each person's admission. How much did it cost in all for them to go to the movies?" Farida placed five 7.00's in a column and added them up. She computed the answer as 35. She then added 35 to another 35 for 70.

I gave Farida the counter suggestion that "someone told me that when you multiply any number by 10, the answer is that number with an extra zero because there are no ones when you multiply by ten." She smiled, and nodded her head. I then gave her the problem, "Eight children received ten dollar bills as birthday presents. If each child gets $10, how much money did they receive in all?" Farida used the exact same strategy that she used for the movie problem, ignoring my suggestion as it apparently made no sense to her.

During the five days, I worked with Farida on problems, discussing with her the strategies she used. Farida was very comfortable adding numbers in the tens, and even the hundreds, but when it came to numbers in the thousands, she would become confused. She demonstrated a good understanding of place value to the tens. She was fluent with her addition facts to ten, and had a sound strategy for finding her facts to 20. In multiplication, she had memorized her facts to 10, but often used addition to find the answer when she could not remember.

My big question was, "Is there a way to help Farida move into multiplicative thinking?" We had spent time in the fall working on multiplication. I had used several units of instruction and a menu of activities designed from a Marilyn Burns model. Although Farida had successfully completed all the work in the units and the menu, she was more comfortable using addition to solve her problems. She had not yet made the transition to multiplicative thinking. I knew she was probably not alone.

Roaming The Known and Group Work

Meeting with our Math Research group, I wasn't sure where to go from here. The group as a whole was struggling with "next steps" and so we decided to "roam the known" with several more children to see what comparisons we could make.

I chose two students to begin with – both fifth graders. Through previous assessments I knew Marco was one of my least proficient mathematicians, while Matt was one of my most proficient.

In November I had assessed both students with similar activities. On the Kamii assessment, Marco scored 18 out of 46 possible answers, while Matt scored a perfect 46. Marco was able to correctly compute 24 out of 100 multiplication facts in a 10 minute test, while Matt scored 100 out of 100. In place value, Marco had little if any concept of tens, while Matt demonstrated place value understanding into the millions. In ranking my 14 fifth grade students by proficiency, with one being the most proficient, and 14 being the least, I had ranked Marco 13th and Matt 2nd.

It was now March, and "Roaming the Known" would hopefully provide me with additional information. I worked with Marco and Matt for about 20 minutes each, using the same activities for both.

Marco was able to estimate about 40 cubes and count them successfully. He was able to identify the one's place, but was unclear about tens and above. His primary strategy for problem solving was counting on his fingers. His father had taught him the algorithms for addition, subtraction, and single digit multiplier multiplication, and he was successful in doing these calculations although he couldn't explain the steps.

When given the word problem, "Twelve people are going to the movies. It costs each person $3 admission. How much money do they need in all?", Marco made 12 tick marks on his paper. He then counted up three on his fingers as he marked off each tick mark. He found the answer 36. When he attempted to find the total of all the numbers on all the cards, the task described in Chapter 5, he recorded all the numbers card by card then tried to add. He did get 340. Marco had memorized the times tables and was comfortable with the facts. When he couldn't remember one, he would count on his fingers to find the correct answer.

Matt worked on the same type of activities Marco. Matt was a confident mathematician and seemed to have an inherent understanding of number. He could identify and explain place value to the millions and used multiple strategies for all problems. When given mental math addition problems, he preferred to work left to right, and could easily take numbers apart and put them back together when calculating. On the cards task, Matt simply totaled the values for one suit then multiplied the result by 4.

When working multiplication problems, Matt easily slid from one strategy to another, depending on the problem and his mind set. When given the problem, "18 x 35" he clustered the problem with "10 x 30, 10 x 5, 8 x 30, 8 x 5". When asked for another strategy, he rounded off the 18 to 20, multiplied 20 x 35, and then subtracted 70. Matt's multiplicative thinking was highly developed and he used multiplication in problem solving with ease.

After these two assessments, I realized how vast the differences are in the way these two students and Farida approach their problem solving. Their strategies seem to be embedded in their understanding of how number works. And no matter how many units I exposed them to, their knowledge of number was the underlying factor that effected their success.

In meeting with my colleagues, I found similarities between Marco and the first graders that Amy and Catherine had assessed. Farida looked very similar to high third graders that Judy had assessed. And, of course, Matt was very similar to my other highly proficient mathematicians - some of whom were fourth graders and some of whom were fifth graders.

In looking at these students, I began examining their behaviors to try and determine the developmental levels they were exhibiting. Marco was a clear candidate for group work activities, while Farida might just need more experience in number to make that jump. Matt also needed to be challenged to expand his number understanding further.

Overall, however, the idea of working with a small group interested me a great deal because of what my colleagues were finding in the primary grades. I wondered, "Were children who had difficulty with number sense 'born' that way, or 'made' through educational experiences? Could Marco and other similar students develop number sense through appropriate activities? Should they be taught the standard algorithm as a tool to 'get by'?"

The research group decided to pursue groups and, along with my colleagues, I chose students who I would try to accelerate with activities that would help them develop the number sense that is so crucial to their understanding of mathematics.

The students I chose were Marco and Cressie, another 5th grader who seemed to be developmentally on the same level with Marco. I was very concerned for their success in middle school next year if they did not have some sense of number. I created a second group of fourth graders that I worked with as well, but will concentrate on the fifth grade group in discussing my findings.

We began meeting together three to four times a week for about 30 minutes. We would begin our sessions with a mental math, followed by a story problem, and then finish with a math game.

Using the information Amy and Catherine had found with their primary students, we began with simple mental math problems like 27 + 8. Both students used counting on strategies in the beginning sessions. They did not look for tens, even when counter suggestions were made. Our math games were any game that asked the students to make doubles and then make tens. My goal was for them to know the 10's combinations automatically and then transfer that knowledge to their calculations.

The progress was slow, but eventually I began seeing more sophisticated strategies. In solving the problem, 27 + 8, Cressie told Marco, "Take the 7, add a 1 and then double that with the 8. That makes 16. You add that to 20 for 36 and then take 1 away."

By the end of the two months, Cressie and Marco showed more confidence with their mathematical skills. They were able to identify place value to the tens, and were able to use multiple strategies for their calculations.

On the Kamii test administered in November, Marco scored 18 out of 46. Cressie did not finish the Kamii test in November because he said it was "too hard." When reassessed in June, Marco scored 42 out of 46, and Cressie not only finished the test, but scored 39. Neither demonstrated multiplicative thinking, but each of them knew their addition facts to 20 and had memorized their multiplication facts to ten's.

Following these sessions with Marco and Cressie, I am convinced that ultimately students can be "recovered" but they must be given opportunities and a great deal of time to develop their understanding. They cannot skip ahead if they do not understand the previous concept. Each concept builds on the next, almost like a pyramid. Equipped with this information, I will completely change my approach to teaching math next year, taking into account all I have learned from this research experience.

Oh yes, I don't want to forget Farida. Wouldn't you know it, by the end of the year, Farida began using multiplicative strategies in her problem solving. On the cards task described in Chapter 5, Farida used addition in the fall, adding up all the columns of 2's, 3's, 4's, etc., then finding the total of the sums. She got 330. In the spring she set the problem up as a multiplication (4 x each number) problem then totaled the tens separately from the other results and came up with 340. So she improved in her organization as well as in her use of multiplication. She is now very comfortable with single digit multipliers, but when confronted with a double digit multiplier, she often reverts back to simpler strategies to solve the problem. Her number sense continues to grow and she sees herself as a good mathematician. So do I.

What Did I Learn?

In summarizing what I learned, I would say the following:

1. Students really do learn at all different levels, and determining their needs is an imperative step in planning instruction.

2. Prior assessment should not only drive instruction, but also serve as a benchmark for success once instruction is completed.

3. When you use a variety of assessments, most of them will show you similar results in terms of ranking students.

4. You can teach some mathematics to the whole class, but opportunities for individual development are important for growth. Menus and games and occasional homogeneous groups are very important for fourth/fifth grade teaching.

5. Students learn number concepts in a hierarchy, with new concepts building on the understanding of previous ones. If one concept is not learned, then it is very difficult for students to continue in their understanding of more complex ones.

6. Students can construct concepts that they missed, but the activities must be appropriate to the level they are starting from. If students do not understand place value to tens, they need to develop that number sense before beginning more complex concepts.

Part V

Judy Kysh

Counting All the Cards: Jeremy's Question

The question first came up in Amy Kari's 1/2/3 multi-age classroom, when Jeremy, a third grader, wondered, "What would all the cards in a whole deck add up to?" When Amy brought the question to our teacher-research group, we immediately agreed it was a good one to ask of all students.

Our teacher research group had been looking at methods of assessment in relation to a list of questions which included the following two.

> What is the role of speed and efficiency with counting, addition, or multiplication in the identification of good elementary mathematicians?
>
> What are some indicators of beginning understanding and use of multiplication? And what problems can we choose to encourage and assess the development of multiplicative thinking?

During the first week in December we gave the problem to the students. How much will you get if you add up all the numbers on all the cards in a regular deck of playing cards? Most students agreed that the Jacks, Queens, and Kings should count as tens, but some older students did the problem two ways, including a second solution with the Jack worth 11, Queen worth 12, and King worth 13. Aces were ones. Students worked alone or in pairs and shared ideas, but each wrote his/her own paper or, in the case of some first graders and one third grader who did it entirely in his head, explained their method as the teacher tape recorded it. All of the children were able to work on the problem and accomplish something, many spending an entire hour even though they did not complete it. Strategies ranged from spreading out the cards randomly and counting spots with tally marks to sophisticated and efficient combinations of problem solving strategies and skills such as combining pairs to make tens and then multiplying.

To sort the papers we compared solution strategies and started making piles of similar solutions. We were particularly interested in the problem solving strategies students used and in whether they would use counting, addition, or multiplication. When we first sorted all the papers into categories based on solution strategies we were surprised by the variety of methods of solution and particularly by some we had not anticipated. We identified ten strategies, then we compared the different strategies, and tried to rank them based on sophistication of organization, efficiency of method, and a "why didn't I think of that?" sense of elegance of solution. After ordering the identified strategies we looked at the papers again and recorded the grade level for each student who had used that strategy.

Categorizing Students' Responses

The graph in *Figure 1* shows the list of categories, in order, based on our assessment of levels. Each number represents the grade of one student. We found that the strategies varied in three major ways:

- Operations used: counting, addition, and multiplication;
- Problem solving strategies used: mainly subproblems, organized lists, or random recording of cards drawn from the deck;
- Level of abstraction (or distance from the actual deck of cards): from counting the spots on the card to drawing and recording numbers from cards to imagining the organization of the deck with its four suits of thirteen cards.

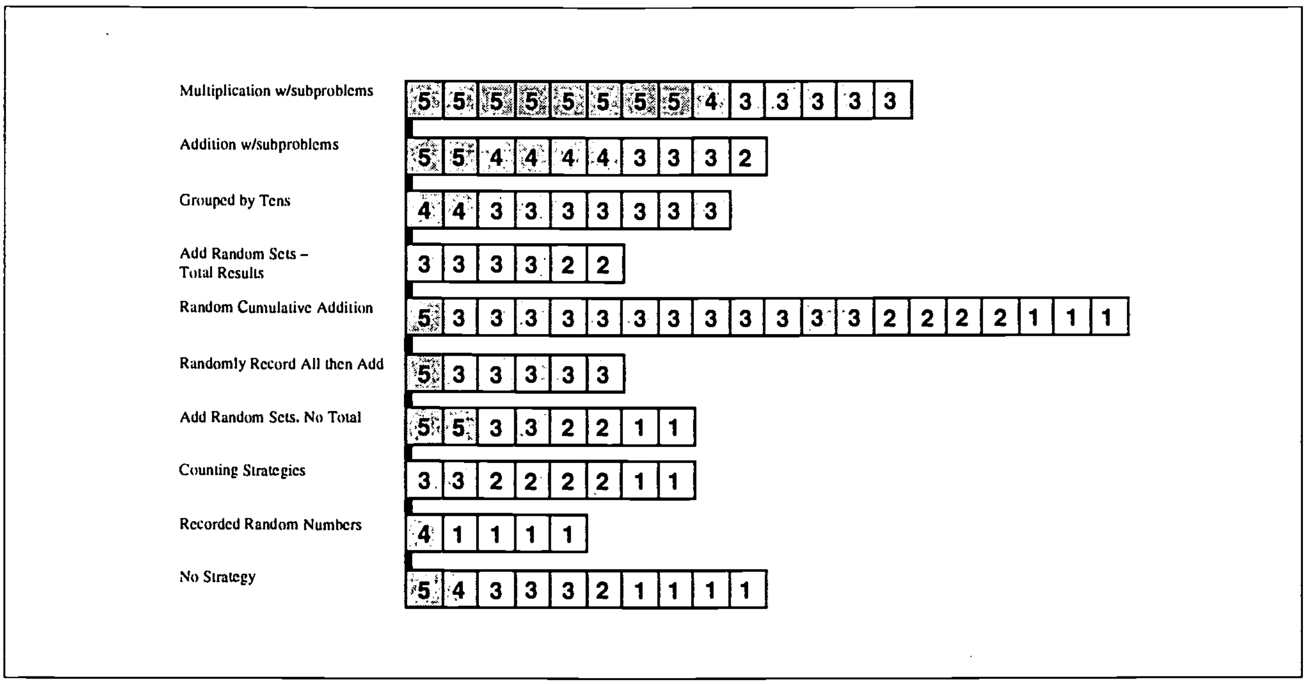


Figure 1: Fall

Some students demonstrated both good skills and good problem solving; others organized well but did not yet have the accuracy and efficiency with number skills to finish; a few were able to complete the problem in spite of a cumbersome strategy; and some showed neither skills nor good problem solving strategies. Marcos drew cards, recorded each card, and then added the whole bunch, and succeeded in getting the correct total. This is not a great strategy for a fifth grader, but he can add. Samantha, a first grader, (see Figure 2) organized her subproblems, four of each kind of number, in order, and counted, but she did not complete the totaling because the counting just took too long.

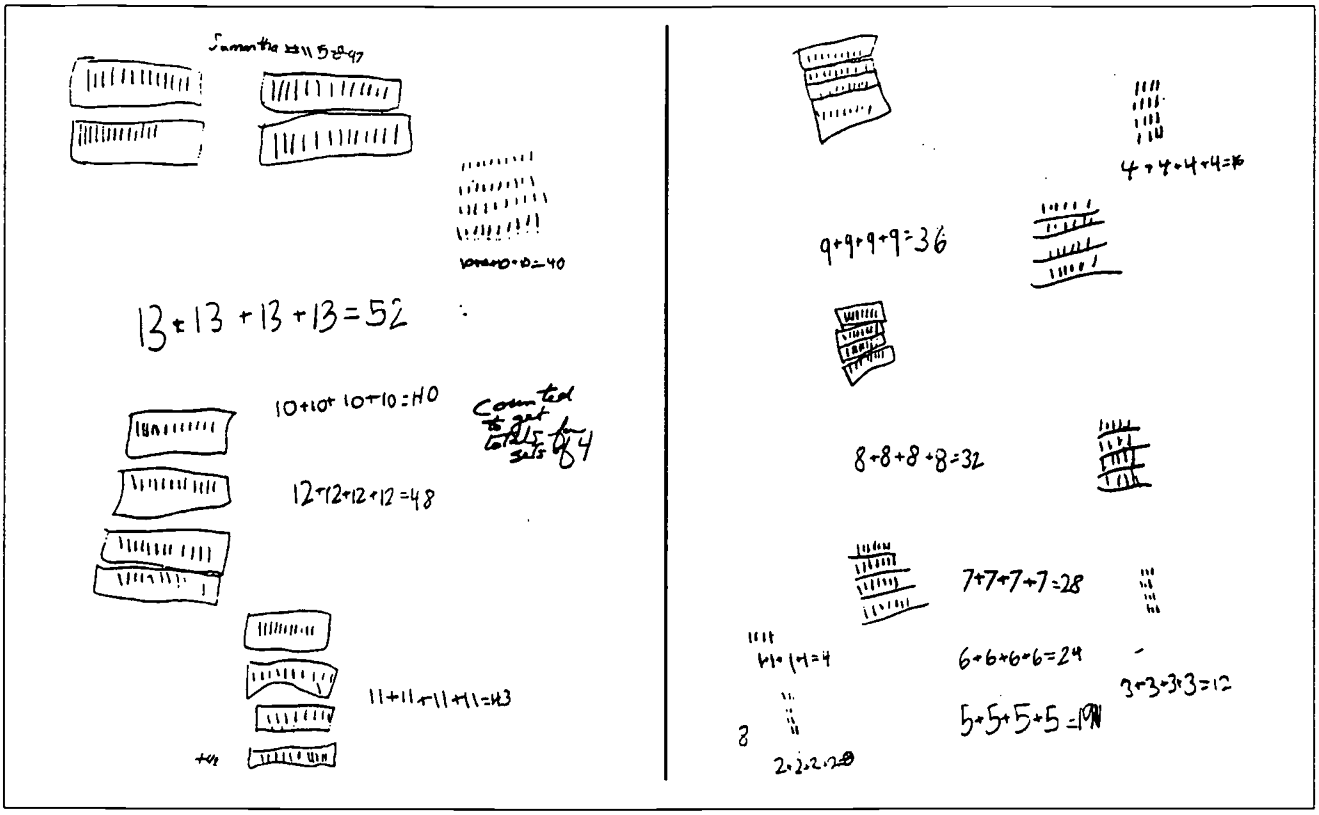


Figure 2: Samantha

Marcos' and Samantha's papers give us a glimpse of the value of this problem for assessment. In this one solution teachers saw an accurate snapshot of each student's progress in relation to both skill development and problem solving. Marcos was capable with addition but often did not stop to see how he might organize a problem to take advantage of what he knows. His strategies are similar to those of most second graders, and, more importantly, with his strategy the question of using multiplication does not arise. Samantha was just beginning to learn addition, but clearly showed potential as a problem solver. No other first graders, except Mary who worked with Samantha, organized the problem into subproblems. So sometimes the level of sophistication in problem solving can far exceed the level of skill development.[1] Ideally they will come together and increase by grade levels, and in general that is what we found (*figure 1*).

The strategies are further described below and representative samples of student work are shown in Figures 2-10.

[1] Samantha did sort the deck of cards to organize her subproblems. It will be interesting to follow her progress over the next couple of years.

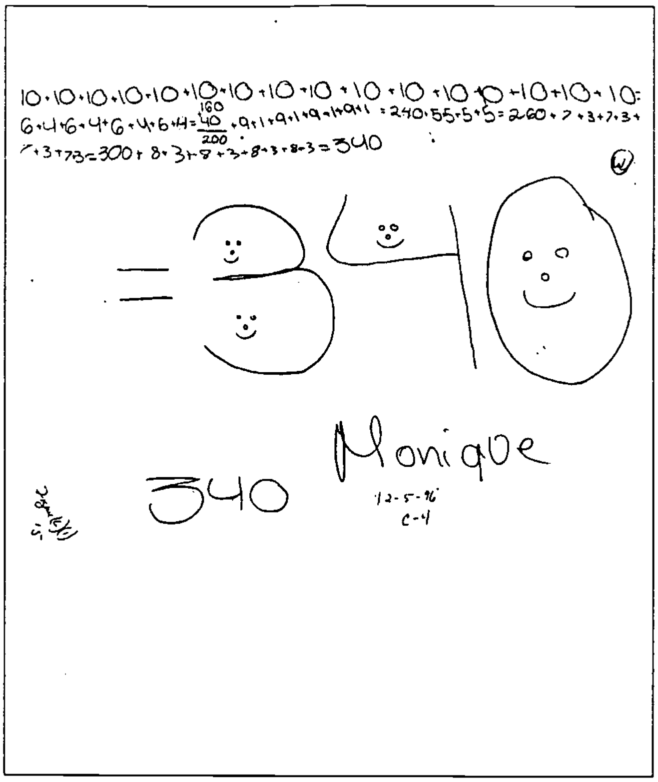


Figure 3: Monique (fall), Group by Tens

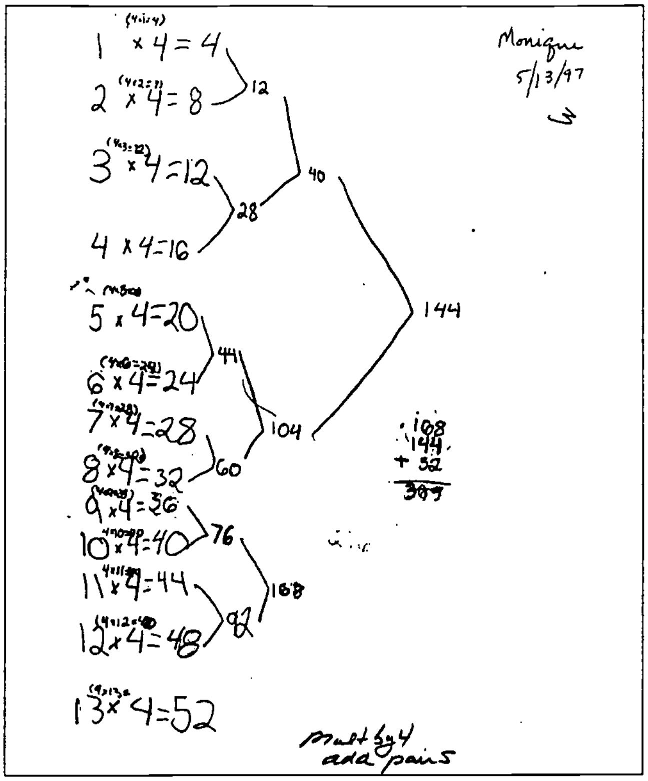


Figure 4: Monique (spring), Multiplication w/Subproblems

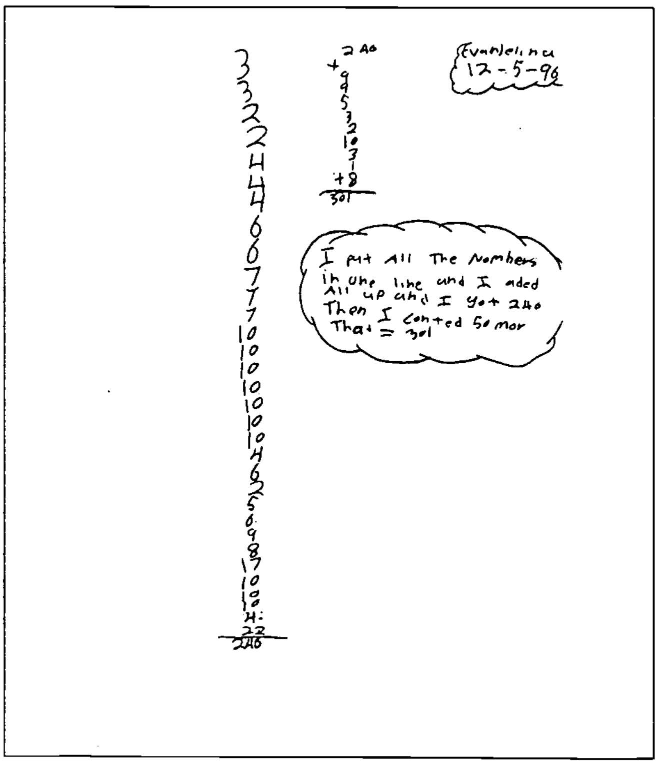


Figure 5: Evanjelina (fall), Randomly Record All then Add

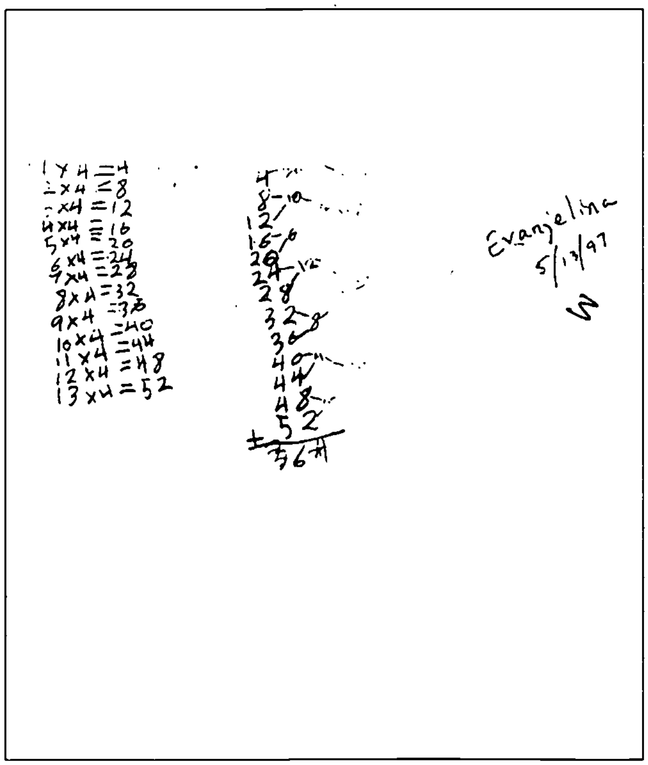


Figure 6: Evanjelina (spring), Multiplication w/Subproblems

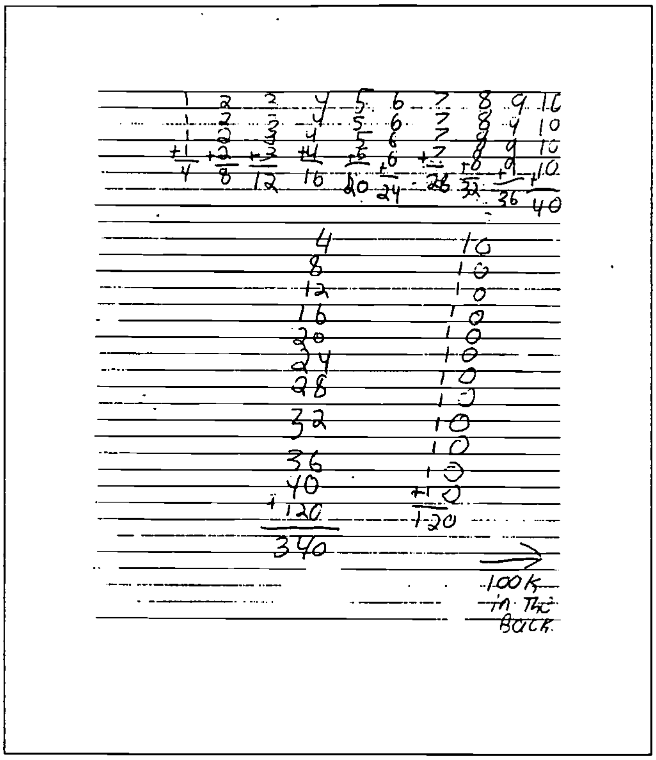


Figure 7: Scott, Addition with Subproblems

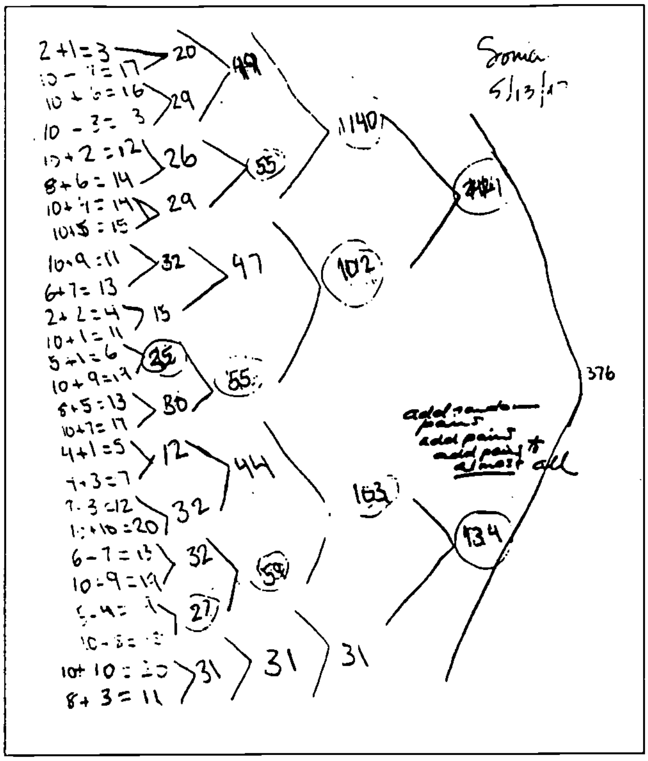


Figure 8: Sonia, Addition of Random Groups

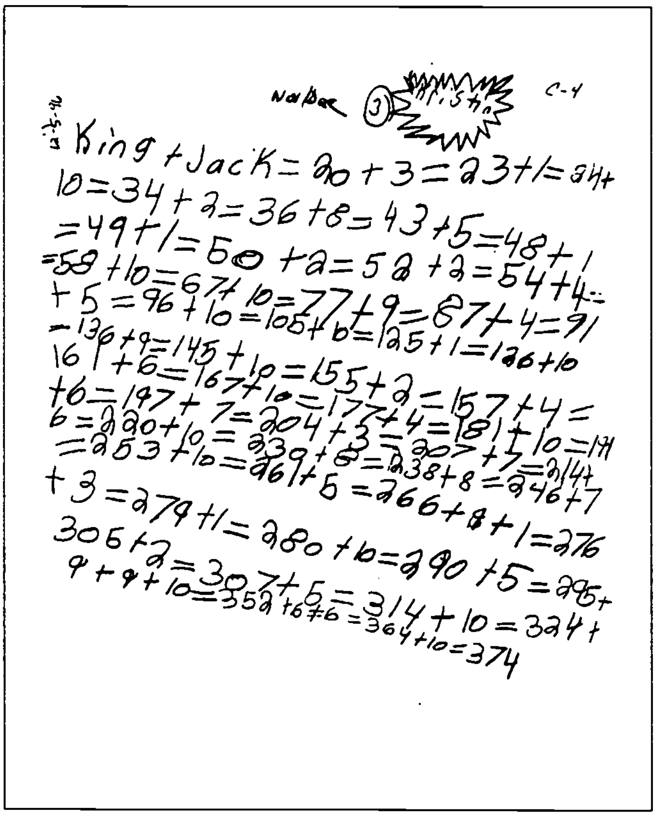


Figure 9: Kristen, Random Cumulative Addition

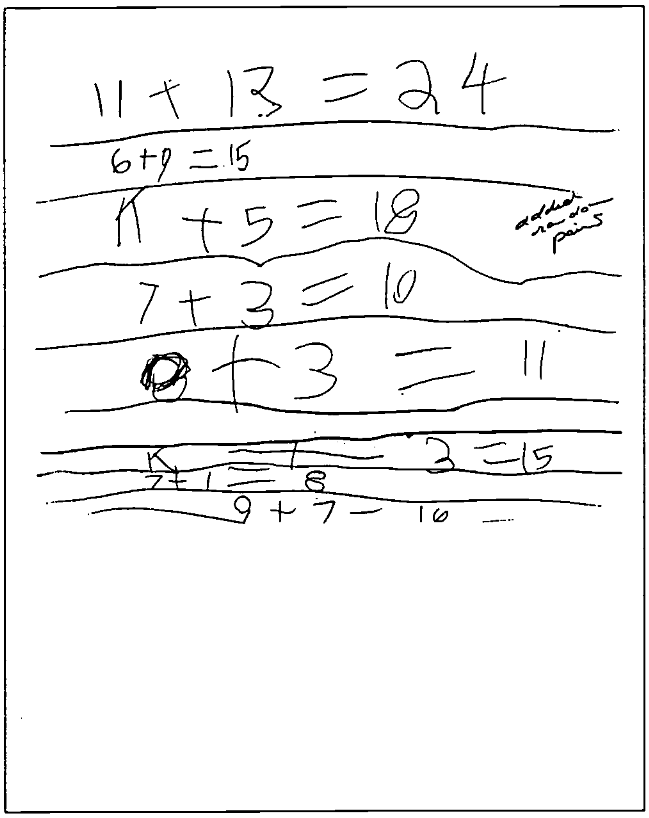


Figure 10: Jourdan, Added Random Sets

Multiplication and Subproblems: The student multiplied each set of four numbered cards by four, or each suit by 4 then added the results (See Monique and Evanjelina, *Figures 4 and 6*). Or the student grouped by tens then multiplied.

Addition with Subproblems: Scott grouped sets of four, 4 ones, 4 twos, 4 threes, etc. then found each sum (e.g. 3 + 3 + 3 + 3), then added all the sums or added each suit then added the four results. Students used three methods for organizing the addition of the results: by pairs, cumulative, and totaling all together either using the standard algorithm or adding tens then adding ones. (*Figure 7,* Scott)

Grouped by Tens: In the fall Monique identified pairs with a sum of ten, 1 + 9, 2 + 8, 3 + 7, 4 + 6, 5 + 5, then counted by or added the tens. (*Figure 3*, Monique)

Addition of Random Sets then Total the Results: Sonia drew cards from the deck and recorded pairs or other groupings, added, then added results. (*Figure 8*, Sonia)

Random Cumulative Addition: Some first added the tens, then added randomly drawn individual cards. Kristin just started with any card added on the next randomly selected card, 7 + 5 = 12, 12 + 3 = 15, 15 + 8 = 23, etc. (*Figure 9*, Kristen)

Random Record All then Add: In the fall Evanjelina turned over cards, wrote all the numbers down in a long column in random order, then tried to add. (*Figure 5*, Evanjelina)

Added Random Sets, No Total: In the fall Jourdan drew cards, recorded pairs or groups, and added them, but made no attempt to total the results. (*Figure 10*, Jourdan)

Counted: Samantha organized cards, recorded and counted, tallied number of spots then counted. Others counted directly from randomly drawn cards with or without recording the numbers along the way. (*Figure 2*, Samantha)

Recorded Random cards: Students just wrote number symbols or tally marks on their papers.

No Strategy: The student copied someone else, usually not very accurately, created new problems with the cards or without; just made up a ballpark answer; gave a totally wild answer; or gave no relevant response.

Comparing Student Responses by Grade Level, Fall and Spring

The graph in Figure 1 shows the grade levels of the students who used each strategy when they did the problem in the fall. We immediately noticed the number of fourth and fifth graders who knew their multiplication facts but did not chose to use them to solve this problem. We were also surprised by the number of third graders who used what we saw as one of the most elegant strategies, combinations to make ten. In fact, when we rank ordered solutions within our original *Addition with subproblems* category Joshua, a third grader, who had the top paper, did the problem mostly in his head. He simply said,

> [In each suit] "There are four cards worth ten, that's 16 tens is 160. Then there are four pairs that make ten, that's 16 pairs and the fives is 18, times 10 is 180. Add 180 and 160 is 340."

In May the students did the problem again. The results are shown in *figure 11* and compared with the fall results in the graph in *figure 12*. Most exciting was the movement of students, especially third graders, into the use of multiplication. Originally we included the category *Group by Tens* as a sub-category of *Addition with Subproblems*, but when we looked at the second set of solutions in May we realized that while many third graders had used the *Group by Tens* strategy in the fall, in the spring they chose instead to add or multiply using subproblems. We decided to separate the two strategies and list the *Group by Tens* strategy below *Addition with Subproblems* because the latter, while it is for this problem more cumbersome, is a more generalizable strategy and leads easily to the next step of multiplying, a step which many students who used *Addition with Subproblems* in the fall took the second time around. For a while, we considered relabeling the *Addition with Subproblems* category as "Pre-multiplicative."

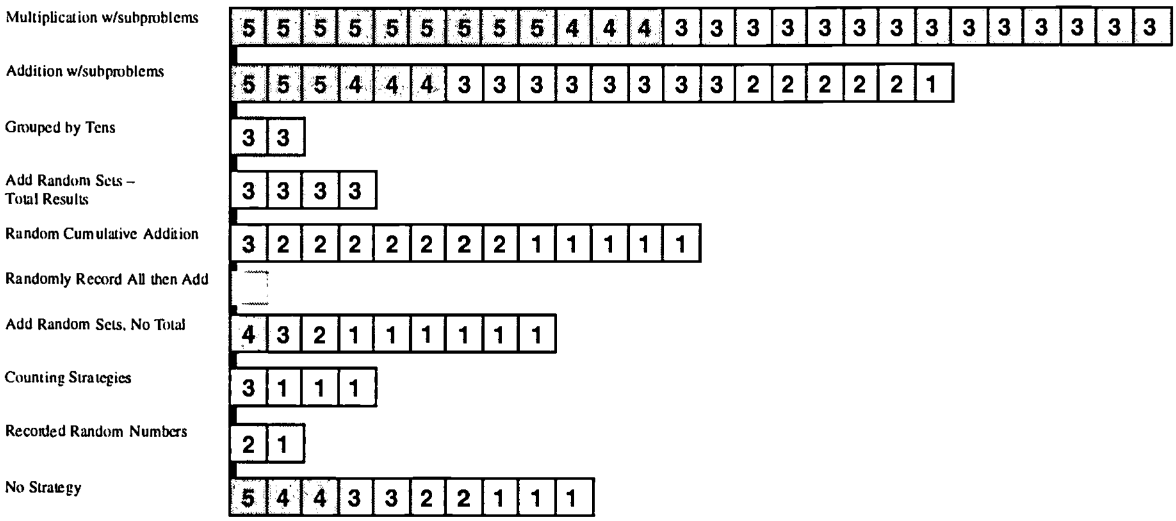


figure 11

In the fall and spring the two groups are not identical because in January the school district decided on smaller class sizes for the primary grades. The third graders in Amy and Catherine's classes were moved to another class. Also a number of second graders apparently did not record their work the first time around, either on paper or tape.

Both times the problem was given there were preferred strategies by grade level, and the grade levels tended to fit the hierarchy of strategies we had outlined, the one exception being the use of combinations to make ten with multiplication. In both the fall and the spring third grade students made the most use of the tens strategy mostly with addition in the fall, some with multiplication in the spring. We wonder whether this strategy would have been selected so often if all the classes had decided on 11, 12, and 13 for Jacks, Queens, and Kings, and whether third graders might have more inclined to use it in December because there is more focus on making tens at that stage in the curriculum.

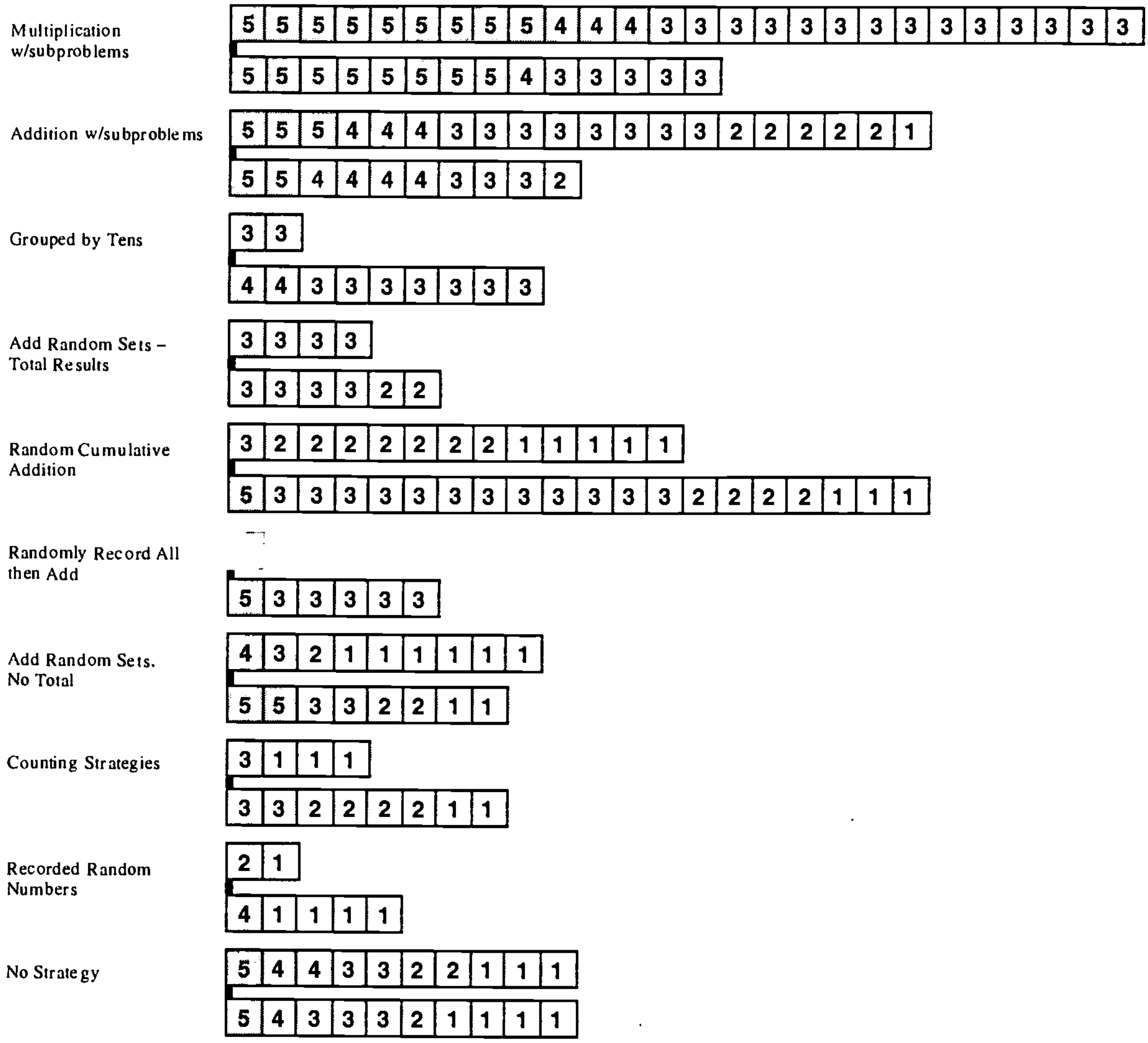


Figure 12

By the end of the year most third graders had abandoned the popular earlier strategies of *Random Cumulative Addition* and *Adding Random Sets* to the second and first graders and moved on to using *Multiplication or Addition with Subproblems*. Also remarkable is the complete abandonment at all levels of the strategy of writing down all the numbers and trying to add them all at once.

When we look at pairs of papers for individual students we found that 66% either moved up in our hierarchy of strategies or improved their completion and accuracy within the same strategy. Twenty percent used essentially the same strategy with no improvement in completion or accuracy, and 14% went backwards. Most of the students who moved backwards were at the extremes. For example, in the fall Joshua (a third grader) used tens and multiplication. In the spring he used tens, but chose to spell out all the steps to show the use of addition and to do the problem twice, first to get 364 using face cards as 11, 12, and 13 then a parallel set up to show how to get 340 using face cards as tens. It is hard to say why he didn't use multiplication, but the way he had carefully laid out his presentation of the problem lent itself more easily to addition (see *figure 13*). Matt and Vincent used multiplication of sets of four by 4 the first time they did the problem; the second time they added the suits then, because the addition of pairs was so easy, they added. In fact, Vincent mentioned that he just added 85 + 85 and then 170 + 170) "because it was easy."

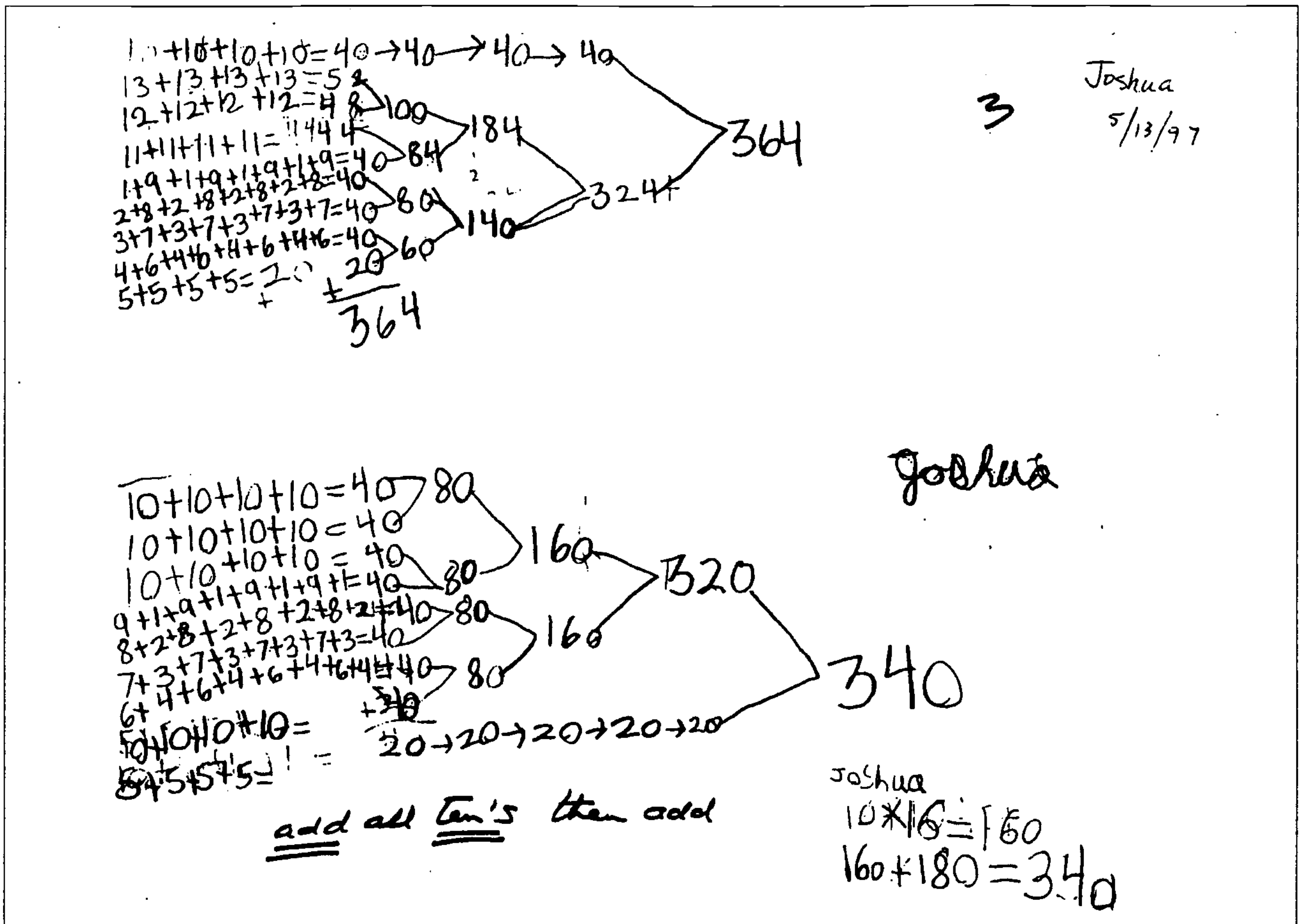


Figure 13

On the other extreme, Derek, a fourth grader recorded random cards and sums the first time around, but the second he added 52, 10, and 1 the three numbers mentioned in his teacher's statement of the problem.

> Find the total value of all the cards in a regular deck of 52 cards. Each face card counts as 10, and each Ace is 1.

We categorized this response as not really relevant to the problem, in other words as *No Strategy.* Judy Kysh, a former high school teacher, thought maybe Derek was reaching a point of frustration which could be leading into a form of resistance; but the teachers, who knew him, assured her that this was Derek's honest try to do the problem. Andrea, a second grader, switched from adding random groups of five cards and attempting to total them to *Cumulative random addition* which is a step lower in the hierarchy, but she improved her accuracy.

For those who improved their solutions the clearest positive shift was in the third graders who moved from using random card-based strategies to multiplication and addition using subproblems. Typical in this group were Monique (Figures 3 and 4) and Evanjelina (Figures 5 and 6). Others moved from addition to multiplication and many more moved from the random card-based categories, *Add Random Sets and Total, Random Cumulative Addition, and Randomly Record All and Add* into the category *Addition with Subproblems.*

In the meantime the second graders moved into *Random Cumulative Addition* and the first graders into *Random Cumulative Addition* and *Add Random Sets, No Total.* Jourdan and Miguel's work show two different ways in which younger students improved their work. Jourdan's first attempt was to count and add cards as she and her partner pulled them from the deck. In the spring she organized subproblems and used addition (see *Figure 14*), a huge step for a second grader. Miguel, on the other hand, stuck with the same strategy, *Random Cumulative Addition*, but improved his speed and accuracy. The first time he only got to 138 with one error; the second he reached 340. In the fall a first and a second grader, DeNisha and Ryon, made up their own problems by drawing random cards and doubling them, while in the spring DeNisha got as far as recording and cumulatively adding eight cards. Progress for most first graders is harder to see because many recorded very little of their work, especially in the fall, but the problem does allow for some first graders to demonstrate beginning problem solving strategies.

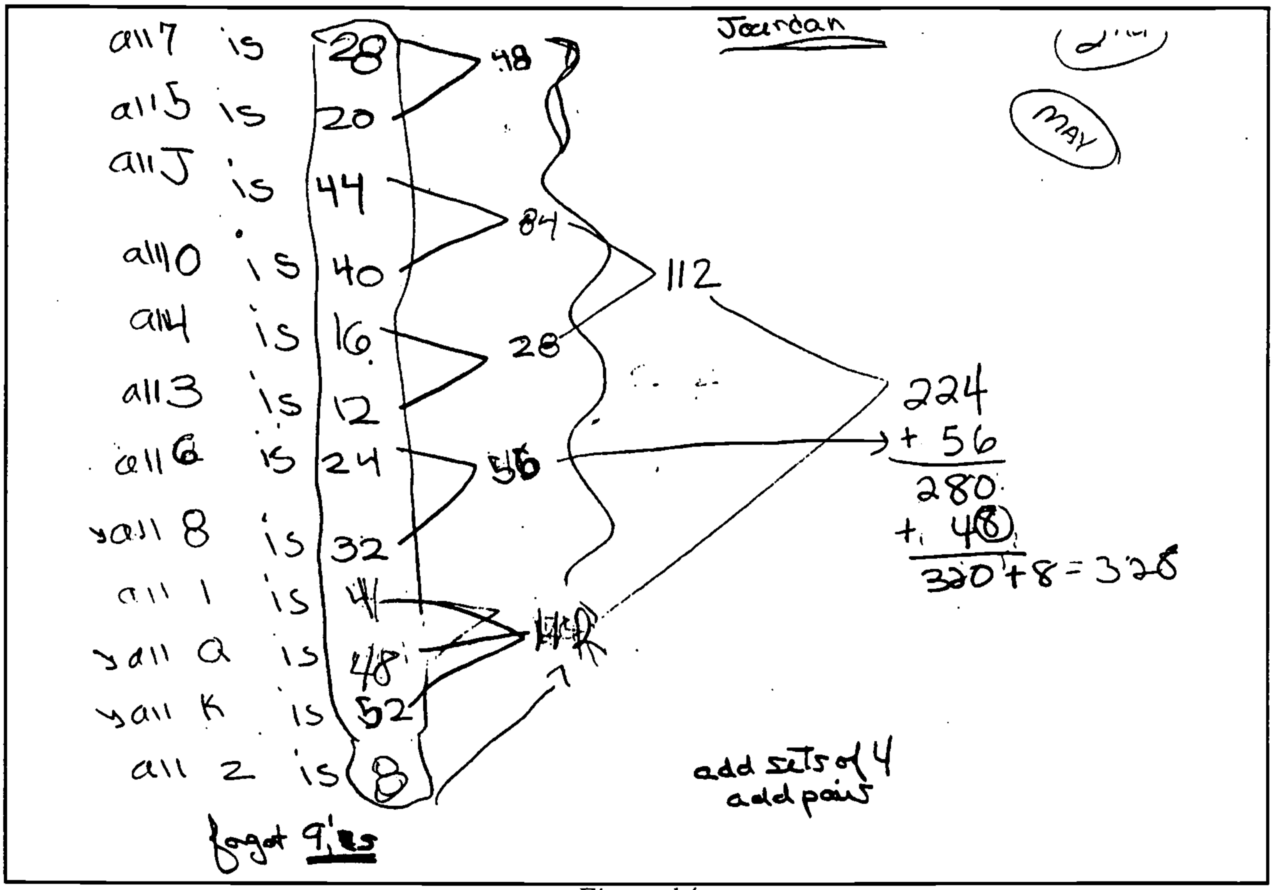


Figure 14

What Did We Learn?

So what did we learn in relation to our original questions? We think this problem has potential for encouraging students' progress toward multiplicative thinking. Among fifth and fourth graders who knew the multiplication facts some used multiplication and some did not. In the spring, more did. This change and the clear change in the third graders strategies toward multiplication and addition with subproblems, the category we began to think of "pre-multiplication," makes this problem look like a good indicator. Moving to the more abstract level of considering the structure of the deck and organizing subproblems to represent it are underlying components in these strategies that are crucial to seeing multiplication as a useful tool. We are seeking more problems like this, not only for assessment but to use in the development of multiplicative thinking.

On the other hand skill with counting, addition, or multiplication certainly makes accurate completion of the problem more likely, and in a few cases made the problem accessible and solvable without an organized strategy. Miguel persevered with *Random Cumulative Addition* and by spring could solve it, but he and two fifth graders were the only ones able to get an accurate result using any of the random, card-based strategies either fall or spring.

The results from this problem show very clearly the importance of both efficiency with skills and good use of problem solving strategies in developing good elementary mathematicians. Skills and problem solving strategies are mutually dependent, and solving this problem not only takes both, but the use of multiplication requires organizing subproblems. Only three children accurately solved the problem using "brute force" and addition skills. And, while the solution is attainable using the tens strategy combined with counting, it was only children already skilled in addition who used that strategy.

And what about Jeremy who posed the question in the first place? His first attempt resulted in some random addition problems and the answer 1020. Unfortunately he did not get a second chance because he was one of the third graders moved to another class. We hope to catch up with him next year when two more teachers (one for third and one for fourth grade) will be joining our teacher-research group to explore these questions further.

REFERENCES

Kamii, C. (1985). *Young children reinvent arithmetic: implications of Piaget's theory.* Columbia University, New York: Teachers College Press.

Steffe, L. (1994). Children's multiplying schemes. In *The development of multiplicative reasoning in learning mathematics*, ed. G. Harel and J. Confrey, 3-40. Albany, NY, SUNY Press.

Vergnaud, G. (1983). Multiplicative structures. In *Acquisition of mathematics concepts and processes,* ed. R. Lesh and M. Landau, 127-174. New York: Academic Press.

Conclusions and Where do we go from here?

Through the time we spent working with individuals and the small groups this year, we began to see similarities that broke barriers of age and expectation. There appear to be some concepts that flow from one grade level to the next. For example without conservation a child cannot move on to place value, and without small number sense a child has difficulty developing multiplicative thinking. When we looked at an exceptional first grader who used multiplicative thinking, he also had conservation, place value and small number sense. However, when we looked at a low performing fourth or fifth grader, these basic concepts had not been constructed yet. These students struggled with their age appropriate curriculum and posed problems for the teacher faced with meeting their needs. Our work this year showed that when we spent time providing these students with appropriately leveled activities and an opportunity to interact socially with a similar peer group, they could make more progress than if they were simply taught rules and procedures.

One result of our continuous discussions of our observations is the following list of common concepts to use for Roaming the Known and also during group Recovery sessions. Sometimes when we were stalled with a child or group the list offered suggestions of where to try moving next, either in a upward or downward direction.

COMMON CONCEPTS USED TO "ROAM THE KNOWN"

* Conserving numbers
* Stones
* Estimation and why it was less accurate in early math learning
* Doubles
* Facts - Dominoes, Dice, Cards for fact (doubles, to 10's, 12's, 20's)
* Books - Number words and symbols
* More/less
* Writing Numbers
* Counting Out-loud (Grouping Single)
* What's a Big Number
* Problems - Explanation of Thinking
* Skip counting - 2's, 5's, 10's
* Fair Shares
* Subtraction
* Multiplication
* Verbal Skills
* Number Systems
* Games for Roaming the Known
    - Domino War
    - Dice War
    - Addition, Multiplication Cards War
    - Other Card Games (Fishing for 10's, 13's, 15's; Aces Out)

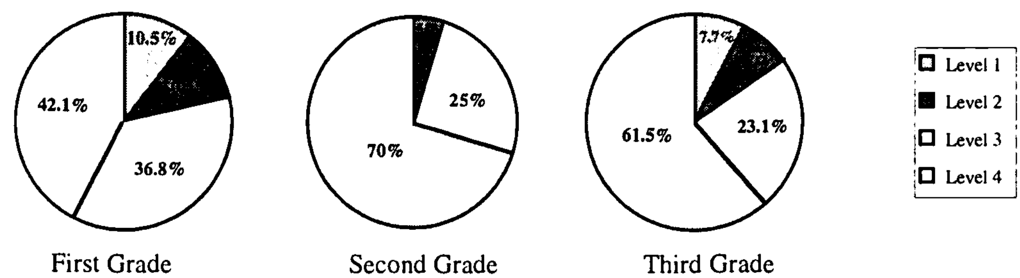
Figure 1

As we found out more and more about individual students we needed a framework to place them in-- What were they supposed to know? This year we were able to develop two rubrics for two of the concepts we felt to be critical at the beginning stages of mathematical development. These rubrics are in draft form and need to be made more detailed for use by others. The first rubric is for our Stones assessment and shows the development of number concept to six which we have decided to use as a year-end benchmark for first grade.

Stones Task: The examiner starts with four stones and asks the student to count them. Then the student is told that the examiner is going to hide some of the stones and the student has to tell how many are hidden. The examiner hides different amounts each time until all the combinations of four are covered. Then the examiner gets five stones and repeats the procedure. Repeat again for six stones.

Below is a graph of the data from grades one through three showing the development of this concept. By the end of first grade, 79% of the students have some level of proficiency with number to six, by the end of second grade only 5% of the students are still struggling with number to six. Our data for third grade shows a larger percentage (15%) of students struggling with number to six. We think this discrepancy occurred because the students in this classroom have not participated in our math program in first and second grade. Only five students had been with the program more than one year.

Stones Assessment by Grade Level



Stones Rubric
Level 1 – Does not have (lower numbers only)
Level 2 – Strategies not always accurate for 6
Level 3 – Has strategies for 6, can accomplish accurately
Level 4 – Fluent, automatic

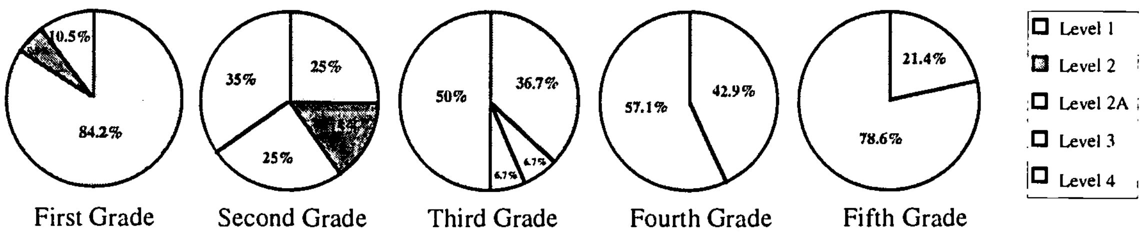
Figure 2

Our second rubric shows the development of place value grades one through five.

Place Value Task: With first and second graders we used a number of cubes or stones (in the twenties) and asked the student to estimate the quantity. Then the child was asked to count the items and the count was recorded. The child was then asked to look at first the digit in the tens column and show it with the cubes or stones. Then the child was asked to show the digit in the ones column. The child's responses were recorded. Upper grade students completed the worksheet shown on page 5 of Appendix A.

A graph showing the data for place value in grades one through five follows in *Figure 3*. The data for place value shows that 84% of our first graders have no understanding of place value. By third grade half the students are able to apply place value to 100 correctly and the number to jumps to almost 79% by fifth grade. These results are substantiated by Constance Kamii in her book *Young Children Reinvent Arithmetic* (p63). This is an important finding because it highlights the slow development of this concept. Furthermore, this development seems to be constant regardless of teacher, teaching methods, or program. In fact, our students progress has come about without the use of any of the traditional place value activities found in the textbooks commonly used in many classrooms.

Place Value Assessment by Grade



Place Value Rubric

Level 1 – No understanding of Place Value up to 100's
Level 2 – Hesitates, questions own response – partial understanding
Level 2A – Responds to counter suggestions
Level 3 – Identifies Place Value up to 100's
Level 4 – Applies Place Value up to 100 correctly

Figure 3

After evaluating our data and experiences for the year, we have decided to redesign our assessment program again. Based on what we now see as more reasonable benchmarks for grades one through five, we are dropping the place value assessment for first graders. We will drop the addition timed test for fourth and fifth grade since 98% of the our third graders scored at 90% or above by year end. The addition test will be replaced by a timed multiplication test. Our new assessment plan is outlined in the table in *Figure 4*.

PROPOSED ASSESSMENTS

1st Grade -	Conservation of Number Stones Pre-Post Addition Facts Test *
2nd Grade -	Place Value Pre-Post Addition Facts Test Beginner Kamii Test Addition Facts to 12, Doubles (Using Dice) *
3rd Grade -	Place Value Pre-Post Addition Facts Test Post Multiplication Facts Test Post Multiplicative Thinking Intermediate Kamii Test Addition Facts to 12 (Using Dice) *
4th Grade -	Place Value Pre-Post Multiplication Facts Test Full Kamii Test (Including Multiplicative Thinking) Multiplication Problems (Single and Double Digit) *
5th Grade -	Place Value Pre-Post Multiplication Facts Test Full Kamii Test - Including Multiplicative Thinking Multiplication Problems (Single and Double Digit) *

* Additional assessments may be given to students who are either way above or way below grade level.

Figure 4

We have also shortened what we call the Kamii test for second and third graders and adapted it to focus on fewer concepts about number. We chose items that would give us the most information about the procedures a child is currently using.

Our work this year with its eventual focus on the recovery of small groups of at risk mathematicians has shown us that this type of intervention can be successful. We are anxious to continue our study and would like to explore the possibility of using a control group from another classroom, and possibly another school, next year. We need to continue to improve our data collection and pre- and post-testing methods.

We anticipate changing our classroom structures to incorporate what we have learned so far. We plan to narrow the focus of instruction in arithmetic, particularly for at risk students, to the common concepts we discovered. To provide the support we were able to give in the recovery groups we see the need for periodic use of homogeneous groups within our heterogeneous classrooms. We think they can be accommodated during regular class time if we develop a system of math menus with leveled activities.

We plan to continue to explore the development of multiplicative thinking, the role of speed and efficiency in effective problem solving, and the behaviors and organizational skills of good mathematicians. We will use activities such as our Cards problem to continue this process.

Appendix A

FALL ASSESSMENT

		1st	2nd	3rd	4th	5th	
Rocks to 6		x	x	x			Report page 4
Timed Test		x	x	x	x		Add or Multiply, A2,A3
Kamii	x	x	x	x			A4
15x23 (place value)			x	x	x	x	A5
Math Rankings		x	x	x	x	x*	
Mat6		x	x	x	x	x	

SPRING ASSESSMENT

	1st	2nd	3rd	4th	5th
Rocks to 6	x	x	x		
Timed Test	x	x	x	x	x
Kamii	x	x	x	x	
15x23 (place value)		x	x	x	x
Mat6	x	x	x	x	x*
Math Rankings	x	x	x	x	x

*Replaced by the CACE test spring. 1997.
*

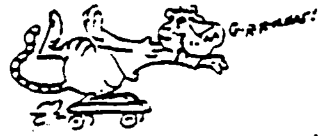
Name # I

100 Addition Facts

10 minute timed quiz

Name ______________

See how far you get in ten minutes.



100 Answers Right______

Add

	a.	b.	c.	d.	e.	f.	g.	h.	i.	j.
1.	0 +5	9 +5	1 +2	6 +2	5 +1	8 +1	0 +6	8 +0	2 +8	8 +4
2.	1 +1	7 +4	2 +0	3 +4	6 +3	1 +8	7 +1	9 +6	3 +5	4 +8
3.	7 +5	0 +4	1 +0	5 +9	2 +9	5 +0	4 +0	8 +2	9 +4	8 +3
4.	2 +1	5 +3	3 +3	9 +7	2 +7	4 +1	0 +8	3 +6	9 +8	0 +0
5.	0 +3	7 +6	2 +2	3 +9	0 +7	7 +9	3 +0	5 +2	9 +3	4 +7
6.	3 +8	6 +0	3 +7	1 +7	4 +6	5 +8	7 +0	1 +3	9 +9	8 +5
7.	2 +3	4 +5	0 +9	6 +9	6 +1	2 +4	0 +2	8 +6	9 +2	5 +7
8.	6 +8	5 +4	6 +7	4 +4	5 +5	7 +3	1 +9	9 +1	2 +6	6 +4
9.	3 +1	7 +7	5 +6	1 +5	4 +2	0 +1	8 +7	4 +3	9 +0	6 +5
10.	7 +8	1 +6	8 +8	2 +5	7 +2	3 +2	1 +4	6 +6	8 +9	4 +9

Drill it Math: The Tiger Book ATA 902

Timed Multiplication Test

Name ______________________________

Find the products.

	[A]	[B]	[C]	[D]	[E]	[F]	[G]	[H]	[I]	[J]
1.	6 × 2	4 × 4	7 × 2	5 × 4	0 × 0	3 × 5	6 × 3	3 × 8	0 × 8	7 × 3
2.	5 × 5	6 × 4	3 × 9	8 × 3	6 × 5	0 × 1	3 × 6	8 × 2	7 × 5	6 × 0
3.	8 × 4	3 × 7	1 × 6	7 × 6	9 × 2	4 × 8	4 × 6	9 × 3	4 × 7	5 × 0
4.	0 × 3	5 × 8	9 × 4	5 × 7	2 × 1	9 × 5	5 × 6	1 × 5	5 × 9	9 × 8
5.	7 × 7	7 × 9	8 × 8	6 × 6	8 × 7	1 × 2	9 × 6	9 × 9	8 × 6	1 × 9
6.	9 × 1	2 × 5	1 × 1	3 × 4	3 × 0	1 × 3	8 × 0	8 × 1	0 × 9	8 × 9
7.	9 × 0	5 × 1	3 × 2	4 × 0	2 × 2	7 × 1	6 × 8	2 × 6	6 × 7	0 × 5
8.	8 × 5	4 × 1	2 × 8	9 × 7	0 × 6	1 × 7	2 × 9	7 × 4	0 × 2	6 × 9
9.	1 × 0	5 × 2	3 × 3	2 × 4	4 × 9	7 × 0	4 × 2	4 × 3	1 × 4	2 × 3
10.	0 × 7	6 × 1	7 × 8	5 × 3	2 × 7	1 × 8	4 × 5	0 × 4	3 × 1	2 × 0

Kamii Test

$4 + 4 =$

$6 + 6 =$

$9 + 9 =$

$8 + 8 =$

$7 + 7 =$

$8 + 2 =$

$3 + 7 =$

$4 + 6 =$

$2 + 6 =$

$5 + 3 =$

$3 + 4 =$

$4 + 5 =$

$5 + 6 =$

$5 + 7 =$

$7 + 8 =$

$7 + 4 =$

$8 + 4 =$

$9 + 4 =$

$8 + 5 =$

$9 + 7 =$

$4 + 1 + 6 =$

$$\begin{array}{r} 6 \\ 3 \\ 7 \\ +2 \\ \hline \end{array}$$

$12 - 6 =$

$10 - 8 =$

$$\begin{array}{r} 22 \\ +7 \\ \hline \end{array}$$

$$\begin{array}{r} 28 \\ +31 \\ \hline \end{array}$$

$$\begin{array}{r} 27 \\ +13 \\ \hline \end{array}$$

$27 + 82 =$

$28 + 72 =$

$$\begin{array}{r} 195 \\ +\ 65 \\ \hline \end{array}$$

$$\begin{array}{r} 299 \\ +301 \\ \hline \end{array}$$

$$\begin{array}{r} 448 \\ +274 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ 52 \\ +186 \\ \hline \end{array}$$

$7 + 52 + 186 =$

$$\begin{array}{r} 48 \\ -27 \\ \hline \end{array}$$

$$\begin{array}{r} 27 \\ -\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 504 \\ -306 \\ \hline \end{array}$$

$612-513=$

$3 \times 7 =$

$4 \times 10 =$

$$\begin{array}{r} 3 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} 10 \\ \times 4 \\ \hline \end{array}$$

$6 \times 6 = 36$
$7 \times 6 =\ ?$

Place Value Test

Name ______________________ Date ____________

PART 1 - Draw 16 dots in each box below. Circle the number of dots each circled number represents.

16 16

PART II - Solve each problem below. Explain your strategies in words and numbers

3 x 75

15 x 23

SE001524

ERIC®

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)

REPRODUCTION RELEASE

(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: What Makes A Good Elementary Mathematician?

Author(s): Catherine Essary, Judy Rummelsburg, Amy Kari, Connie Gillan, Judy Kysh

Corporate Source:

Publication Date:

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, *Resources in Education* (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic media, and sold through the ERIC Document Reproduction Service (EDRS). Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following three options and sign at the bottom of the page.

The sample sticker shown below will be affixed to all Level 1 documents	The sample sticker shown below will be affixed to all Level 2A documents	The sample sticker shown below will be affixed to all Level 2B documents
PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY Sample TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) 1	PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE, AND IN ELECTRONIC MEDIA FOR ERIC COLLECTION SUBSCRIBERS ONLY, HAS BEEN GRANTED BY Sample TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) 2A	PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN MICROFICHE ONLY HAS BEEN GRANTED BY Sample TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC) 2B
Level 1	Level 2A	Level 2B
[X]	[ ]	[ ]
Check here for Level 1 release, permitting reproduction and dissemination in microfiche or other ERIC archival media (e.g., electronic) and paper copy.	Check here for Level 2A release, permitting reproduction and dissemination in microfiche and in electronic media for ERIC archival collection subscribers only	Check here for Level 2B release, permitting reproduction and dissemination in microfiche only

Documents will be processed as indicated provided reproduction quality permits.
If permission to reproduce is granted, but no box is checked, documents will be processed at Level 1.

I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche or electronic media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries.

CRESS CENTER
ONE SHIELDS AVE
UNIVERSITY OF CALIFORNIA
DAVIS CA 95616
jmbush@ucdavis.edu

Sign here, → please

Signature: Judy Rummelsburg Judith Kysh	Printed Name/Position/Title: Judy Rummelsburg, Judith Kysh
Organization/Address: UC Davis/Rio Vista Elementary	Telephone: (530) 752-8393 FAX: (530) 752-6135
	E-Mail Address: JRCELLO@AOL.Com Date: 4/15/98

(over)

III. DOCUMENT AVAILABILITY INFORMATION (FROM NON-ERIC SOURCE):

If permission to reproduce is not granted to ERIC, *or*, if you wish ERIC to cite the availability of the document from another source, please provide the following information regarding the availability of the document. (ERIC will not announce a document unless it is publicly available, and a dependable source can be specified. Contributors should also be aware that ERIC selection criteria are significantly more stringent for documents that cannot be made available through EDRS.)

Publisher/Distributor:
Address:
Price:

IV. REFERRAL OF ERIC TO COPYRIGHT/REPRODUCTION RIGHTS HOLDER:

If the right to grant this reproduction release is held by someone other than the addressee, please provide the appropriate name and address:

Name:
Address:

V. WHERE TO SEND THIS FORM:

Send this form to the following ERIC Clearinghouse:

THE UNIVERSITY OF MARYLAND
ERIC CLEARINGHOUSE ON ASSESSMENT AND EVALUATION
1129 SHRIVER LAB, CAMPUS DRIVE
COLLEGE PARK, MD 20742-5701
Attn: Acquisitions

However, if solicited by the ERIC Facility, or if making an unsolicited contribution to ERIC, return this form (and the document being contributed) to:

ERIC Processing and Reference Facility
1100 West Street, 2nd Floor
Laurel, Maryland 20707-3598

Telephone: 301-497-4080
Toll Free: 800-799-3742
FAX: 301-953-0263
e-mail: ericfac@inet.ed.gov
WWW: http://ericfac.piccard.csc.com